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(10+2)

All PHYSICS DERIVATION

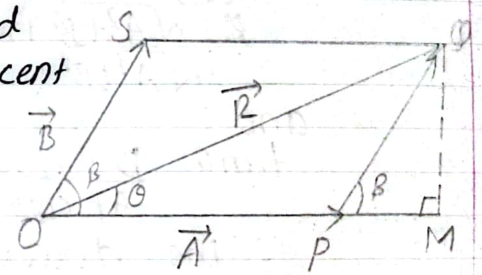
Mechanics
Chapter-2

'Vector'

1. Parallelogram law of vector addition

→ The law states that, "If two vectors acting simultaneously from a point are represented both in magnitude and direction as adjacent sides of parallelogram; then the diagonal of the parallelogram passing through that point represents resultant both in magnitude and direction.

→ As shown in fig. $\vec{A}$ and $\vec{B}$ represent two adjacent sides of parm. & $\vec{R}$ represents resultant as diagonal OQ.



OP is produced upto M such that $QM \perp OM$.

β = angle between $\vec{A}$ and $\vec{B}$ & θ = angle between $\vec{R}$ & $\vec{A}$

From rt. angled ΔPMQ,

$\sin\beta = \frac{QM}{PQ} \Rightarrow QM = B\sin\beta$

$\cos\beta = \frac{PM}{PQ} \Rightarrow PM = B\cos\beta$

Now, In rt. angled ΔOMQ

$OQ^2 = OM^2 + QM^2$

Or, $R^2 = (OP+PM)^2 + (QM)^2$

Or, $R^2 = (A + B\cos\beta)^2 + (B\sin\beta)^2$

Or, $R^2 = A^2 + 2AB\cos\beta + B^2\cos^2\beta + B^2\sin^2\beta$

Or, $R^2 = A^2 + 2AB\cos\beta + B^2$

Or, $\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\beta}}$ —(i)

Now,

$\tan\theta = \frac{QM}{OM}$

or, $\tan\theta = \frac{B\sin\beta}{A + B\cos\beta}$

or, $\boxed{\theta = \tan^{-1}\left(\frac{B\sin\beta}{A + B\cos\beta}\right)}$ —(ii)

Thus, the magnitude and direction of resultant is known from (i) & (ii) resp.

Special cases:-

* When $\theta = 0°$ i.e. When two vectors act parallel, then we have

$$R = \sqrt{A^2+B^2+2AB\cos 0}$$
$$= \sqrt{A^2+B^2+2AB}$$
$$\therefore R = A+B \quad (\text{Maximum})$$

and $\tan\theta = \dfrac{B\sin\beta}{A+B\cos\beta} = \dfrac{B\sin 0}{A+B\cos 0} = 0$

or, $\theta = 0°$

* When $\beta = 90°$; when two vectors act perpendicularly.

Then,
$$R = \sqrt{A^2+B^2+2AB\cos 90}$$
$$= \sqrt{A^2+B^2}$$

and
$$\tan\theta = \frac{B\sin 90}{A+\cos 90} = \frac{B}{A}$$

$\therefore$ The direction of $\vec{R}$ is $\theta = \tan^{-1}\left(\frac{B}{A}\right)$

* When $\beta = 180°$ i.e. When two vectors act anti-parallel.

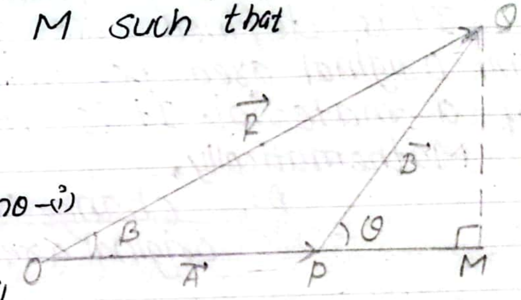
Then, we have
$$R = \sqrt{A^2+B^2+2AB\cos 180°}$$
$$= \sqrt{A^2+B^2-2AB}$$
$$\therefore R = (A-B) \quad (\text{Minimum})$$

and,
$$\tan\theta = \frac{B\sin 180}{A+\cos 180°} = 0$$

2. Triangle Law of vector addition

It states that "If two vectors acting simultaneously at a point are represented both in magnitude and direction by two sides of a triangle taken in same order, then the third side of a triangle taken in the opposite order represents ~~m~~ the magnitude and direction of resultant.

→ Let, $\vec{A}$ and $\vec{B}$ are two vectors acting simultaneously on a particle, represented by OP and PQ of a triangle OPQ and θ be angle between them. Then $\vec{R}$ represents the magnitude and direction of resultant of vectors $\vec{A}$ and $\vec{B}$. P is produced upto M such that QM ⊥ PM.



* ~~Res~~ Magnitude,

In rt-angled Δ PMQ,

$\sin\theta = \dfrac{QM}{PQ} \Rightarrow QM = B\sin\theta$ —(i)

$\cos\theta = \dfrac{PM}{PQ} \Rightarrow PM = B\cos\theta$ —(ii)

Now, $OQ^2 = (OP+PM)^2 + QM^2$

or, $(\vec{R})^2 = (A+B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$

or, $\boxed{R = \sqrt{A^2+B^2+2AB\cos\theta}}$ —(i)

* direction,

Let, β be angle between $\vec{R}$ & $\vec{A}$. Then,

$$\tan\beta = \frac{QM}{OM} = \frac{B\sin\theta}{A+B\cos\theta}$$

$\therefore \boxed{\beta = \tan^{-1}\dfrac{B\sin\theta}{A+B\cos\theta}}$ —(ii)

Thus, Magnitude and direction of resultant is known from (i) & (ii) re

Unit-2 (Heat & Thermodynamics)

#1. Heat and Temperature

3. Relationship between coefficient of linear expansion, superficial expansion and cubical expansion.

→ * Coefficient of linear expansion(α)

It is defined as the change in length per unit original length per unit change in temperature. It is represented by 'α'.

Mathematically,

$$\alpha = \frac{\text{change in length } (\Delta l)}{\text{original length}(l_1) \times \text{change in temperature } (\Delta\theta)}$$

* Coefficient of superficial expansion (β)

It is defined as change in area per unit original area per unit change in temperature of a material. It is represented by 'β'

Mathematically,

$$\beta = \frac{\text{change in area } (\Delta A)}{\text{original area}(A_1) \times \text{change in temperature } (\theta_1)}$$

* Coefficient of cubical expansion(γ)

It is defined as change in volume per unit original volume per unit change in temperature. It is represented by 'γ'.

Mathematically,

$$\gamma = \frac{\text{change in volume } (\Delta V)}{\text{original volume}(V_1) \times \text{change in temperature}(\Delta\theta)}$$

$l_1 = l_0(1+\alpha\Delta\theta)$

$\frac{l_1 - l_0}{\alpha\, l_0} = \Delta\theta$

To derive $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

* Relationship between α and β

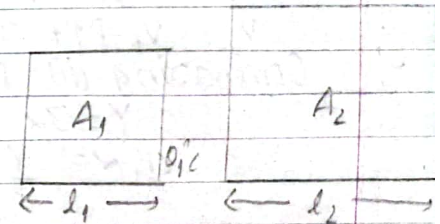
Suppose a square metallic sheet initially ~~with~~ with length l_1 at temperature θ_1°C and with area A_1 is heated to temperature θ_2°C such that length becomes l_2 and area becomes A_2.

Then, from linear expansion,

$l_2 = l_1(1+\alpha\Delta\theta)$ —(i) where, α = linear expansivity

$\Delta\theta = \theta_2 - \theta_1$

We know,



$A_2 = l_2^2$

or, $A_2 = [l_1(1+\alpha\Delta\theta)]^2$ [From (i)]

or, $A_2 = l_1^2[1 + 2\alpha\Delta\theta + \alpha^2\Delta\theta^2]$

Since, value of α is very small we neglect terms having α^2. Then,

$A_2 = A_1[1 + 2\alpha\Delta\theta]$ —(ii) [$\because A_1 = l_1^2$]

From ~~a~~ superficial expansion,

$A_2 = A_1[1 + \beta\Delta\theta]$ —(iii) [β = superficial expansivity]

Comparing (ii) & (iii), we have

$2\alpha = \beta$

or, $\alpha = \frac{\beta}{2}$, which is required relation.

* Relation between α and γ

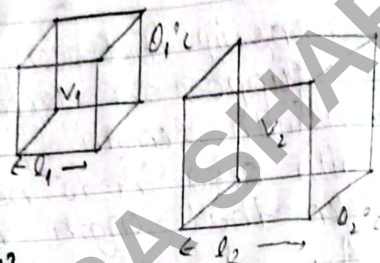
Suppose a cubical metallic sheet at θ_1°C with length l_1 and volume V_1 is heated to θ_2°C such that its length becomes l_2 and volume becomes V_2.

Then,
From linear expansion,

$l_2 = l_1[1+\alpha\Delta\theta]^3$ —(i)

From cubical expansion,

$V_2 = V_1[1+\gamma\Delta\theta]$ —(ii)



We know,

$V_2 = (l_2)^3$

Or, $V_2 = [l_1(1+\alpha\Delta\theta)]^3$ {[From eq^n (i)]}

Or, $V_2 = (l_1)^3[1+3.1^2.\alpha\Delta\theta + 3.1.\alpha^2\Delta\theta^2 + \alpha^3\Delta\theta^3]$

Since, α is very small we neglect terms having α^2 and α^3. Then

$V_2 = V_1[1+3\alpha\Delta\theta]$ —(iii)

Comparing (ii) & (iii) we get,

$\gamma = 3\alpha$

or, $\alpha = \frac{\gamma}{3}$, which is required relation.

Thus, $\boxed{\alpha = \frac{\beta}{2} = \frac{\gamma}{3}}$

4. Method to determine linear expansivity of solid.
[By using Pullinger's apparatus]

→ Apparatus:
Pullinger's apparatus is used to determine linear expansivity of solid(metal). The apparatus consists of a hollow wooden ~~cham~~ frame in which the metal rod is fixed which linear expansivity is to be measured such that the lower end 'B' of rod is fixed at the metallic plate and the upper end 'A' is free to expand. Thermometer is inserted to measure temperature. Spherometer is fixed in the upper part such that the central screw just touches the ground and the connected galvanometer shows deflection.

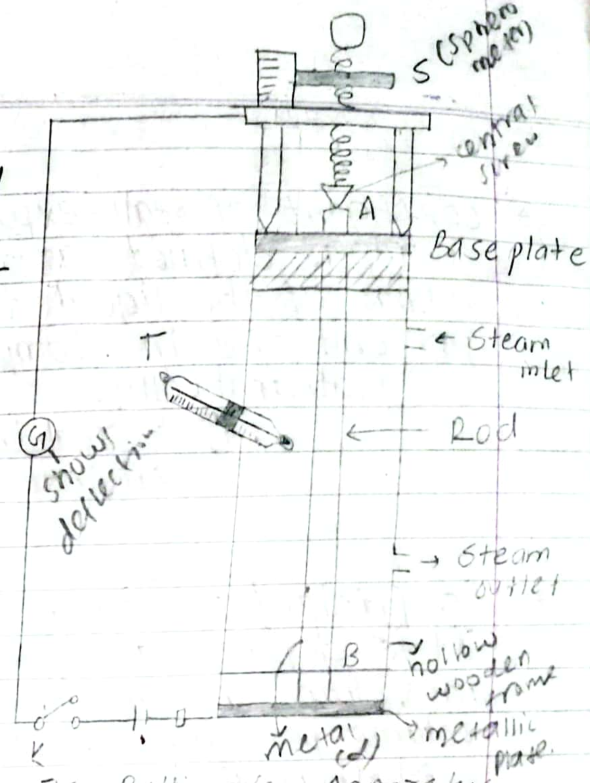


Fig: Pullinger's Apparatus.

Procedure:-
At first, the initial length of rod 'l_1' & initial temperature 'θ_1°C' are noted and the initial reading of spherometer 'R_1' is also written. Then, spheremeter is rotated upward such that there will be space for expansion of Rod. Then steam is passed in the frame till the final steady temperature 'θ_2°C', then the spherometer is rotated downward to note the final reading 'R_2'.

Here, Let,
Initial length of rod = L_1
Initial reading of spherometer = R_1
Initial temperature = θ_1
Final reading of spherometer = R_2
Final temperature = θ_2

Then, the ~~cub~~ linear expansion α of liquid is given by:-

$\alpha = \frac{R_2 - R_1}{L_1(\theta_2-\theta_1)}$

Thus, linear expansivity of rod is determined.

To find cubical expansivity linear expansivity is multiplied by 3.

5. Relation between real and apparent expansion of liquid.

* **Coefficient of real expansion (γ_r)**
It is defined as the real increase in volume of the liquid to the original volume per unit rise in temperature.
Mathematically,

$$\gamma_r = \frac{\text{real increase in volume } (\Delta V_r)}{\text{original volume } (V) \times \text{rise in temperature } (\Delta\theta)}$$

* **Coefficient of apparent expansion (γ_a)**
It is defined as the apparent increase in volume of the liquid to the original volume per unit rise in temperature.
Mathematically,

$$\gamma_a = \frac{\text{apparent increase in volume } (\Delta V_a)}{\text{original volume } (V) \times \text{rise in temperature } (\Delta\theta)}$$

Relation between real expansivity and apparent expansivity.

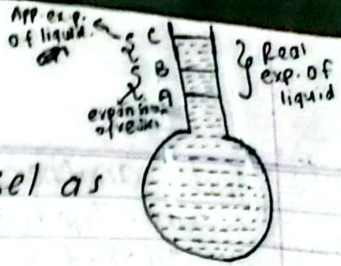
Let us consider a glass vessel of volume V filled with liquid at temperature θ_1. When it is heated to θ_2 it expands and real increase in volume is given by

$$\Delta V_r = \gamma_r V \Delta\theta$$

Similarly, apparent increase in volume is given by

$$\Delta V_a = \gamma_a V \Delta\theta$$

And



increase in volume in vessel as

$$\Delta V_g = \gamma_g V \Delta\theta$$

We know,
Real increase in volume = apparent increase in volume + increase in volume of vessel

Thus, we have

$$\Delta V_r = \Delta V_a + \Delta V_g$$

or, $\gamma_r V\Delta\theta = \gamma_a V\Delta\theta + \gamma_g V\Delta\theta$

or, $\gamma_r = \gamma_a + \gamma_g$

If, α is linear expansion of vessel then,

$$\boxed{\gamma_r = \gamma_a + 3\alpha}$$

, which is the required relation.

6. Determination of real expansivity of liquid by using balanced column. [By Dulong's and petit's experiment]

→ The experimental arrangement for the determination of real expansivity of liquid is shown in the figure. A U-shaped tube is filled with the liquid which real expansivity is to be measured. One side of the tube is covered by Jacket 'X' which contains ice water and stirrer.

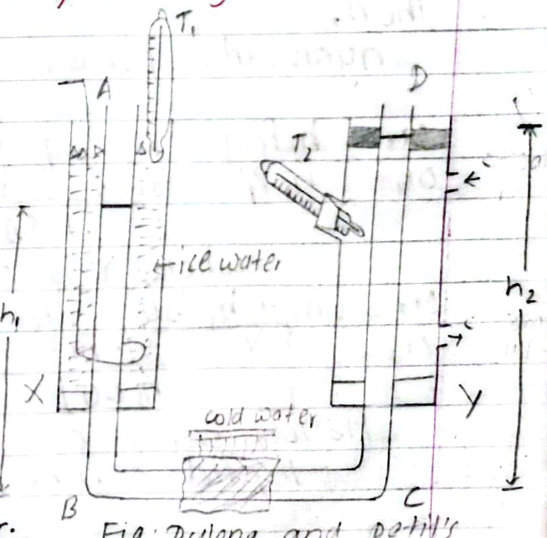


Fig: Dulong and petit's experiment.

And another Jacket Y is supplied with steam. Thermometer T_1 and T_2 are kept in Jacket X and Y respectively to measure temperature. ~~And the horizontal~~ Initially tube on both side will have same level of mercury but as they ~~temp~~ attain thermal equilibrium they have different level. To prevent the flow of heat from hot column to cold column wet cloth is wrapped in BC tube and supplied with cold water continuously.

It is based on hydrostatics which states the height of two liquid column which produce same pressure is inversly proportional to its density.

Let, h_1 be the height of liquid in limb AB and θ_1 be its temperature with density ρ_1 and h_2 be height of liquid in limb CD with temperature θ_2 and density ρ_2.

Then,

hydrostatic pressure at B = hydrostatic pressure at C.

or, $h_1 \rho_1 g = h_2 \rho_2 g$

or, $h_1 \rho_1 = h_2 \times \dfrac{\rho_1}{1+\gamma\Delta\theta}$ $\left[\because \rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}\right]$

where, γ is real expansivity of liquid

or, $h_1 + h_1\gamma\Delta\theta = h_2$

or, $\boxed{\gamma = \dfrac{h_2 - h_1}{h_1 \Delta\theta}}$ —(i)

Hence, using eq^n(i) real expansivity of liquid can be determined.

7. Change in density of substance with change in temperature.

When temperature of the substance is increased then its volume increases but density will decrease as volume and density are inversely proportional.

Let, m and V_1 be the mass and volume of substance at temperature θ_1. The density of substance is given by:

$$\rho_1 = \frac{m}{V_1} \quad \text{—(i)}$$

When the substance is heated to temperature θ_2 its volume becomes V_2 and the density of substance is given by:

$$\rho_2 = \frac{m}{V_2} \quad \text{—(ii)}$$

From (i) and (ii),

$\rho_1 V_1 = \rho_2 V_2$

Or, $\rho_1 V_1 = \rho_2 [V_1(1+\gamma\Delta\theta)]$ [where, γ = cubical expansivity of substance and $\Delta\theta = \theta_2 - \theta_1$]

or, $\rho_1 \cancel{V_1} = \rho_2 \cancel{V_1}(1+\gamma\Delta\theta)$

or, $\boxed{\rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}}$

Thus, density decreases with increase in temperature.

8. Determination of Specific Heat Capacity of a solid by the Method of Mixture. [By a Regnault's apparatus]

→ Specific heat capacity of unknown solid can be determined by the method of mixture. For it, a empty caloriemeter with stirrer is measured as mass of 'm_c' and water is filled upto 2/3 part of caloriemeter and again the mass of ~~caloriemeter with~~ water is taken as 'm_w' with initial temperature 'θ_1°C'. In a steam chamber the experimental solid is heated upto 'θ_2°C' and its mass is taken as 'm_s'. The steam is passed by boiling water. Then, the hot experimental solid is put carefully in the caloriemeter and the ~~...~~ water is stirred till the caloriemeter and solid will have final steady temperature 'θ_3°C'.

The experimental arrangement is shown in the figure:-

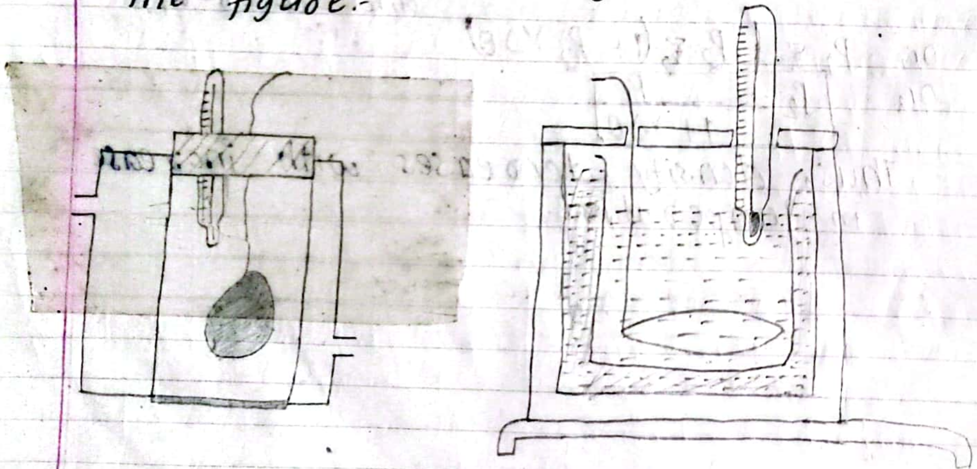


Fig: Regnault's Apparatus

Let, S_c, S_w and S_s be the specific heat capacity of caloriemeter, water and experimental solid respectively.

Then,

From principle of Caloriemetry:

Heat gained by ~~hot~~ cold body = Heat lost by hot body

or, Heat gained by caloriemeter and water = Heat lost by solid

or, $m_c \times S_c \times (\theta_3 - \theta_1) + m_w \times S_w \times (\theta_3 - \theta_1) = m_s \times S_s \times (\theta_2 - \theta_3)$

or, $\boxed{S_s = \dfrac{(m_c S_c + m_w S_w)(\theta_3 - \theta_1)}{m_s(\theta_2 - \theta_3)}}$

Hence, By knowing parameters on right hand side specific heat capacity of unknown solid can be determined.

9. Determination of specific heat capacity of liquid by method of cooling.

* Newton's law of cooling

It states that, "The rate of loss of heat is directly proportional to the difference in temperature of body and its surrounding."

i.e. Rate of loss of heat

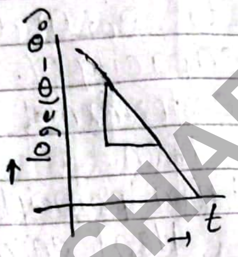
$-\dfrac{d\theta}{dt} \propto (\theta - \theta_0)$, where θ = temperature of body and θ_0 = temperature of surrounding.

[If $\theta > \theta_0$, body losses heat]

Or, $-\frac{d\theta}{dt} = K(\theta-\theta_0)$

or, $-ms\frac{d\theta}{dt} = K(\theta-\theta_0)$

or, $\frac{d\theta}{(\theta-\theta_0)} = -\frac{K}{ms}\,dt$



Integrating both sides, we get

or, $\log_e(\theta-\theta_0) = \frac{-Kt}{ms} + C$

where, C is integral constant.
Above equation is the equation of straight line.

* Specific heat of unknown liquid

This experiment is based on the Newton's law of cooling which states that "When two liquids are cooled under identical condition then the rate of cooling will be same."
Here, Two identical caloriemeters are taken and weighed as 'm_1' and 'm_2' respectively. Then 1st caloriemeter is filled with water of mass 'm_3' and 2nd caloriemeter is filled with experimental liquid with mass 'm_4'. Both caloriemeters are kept inside a box such that they are in identical condition in every respect. Now, both caloriemeters with initial temperature 'θ_1' are allowed to cool upto final steady temperature 'θ_2'. Time interval for cooling of water and liquid is taken as 't_1' and 't_2' respectively.

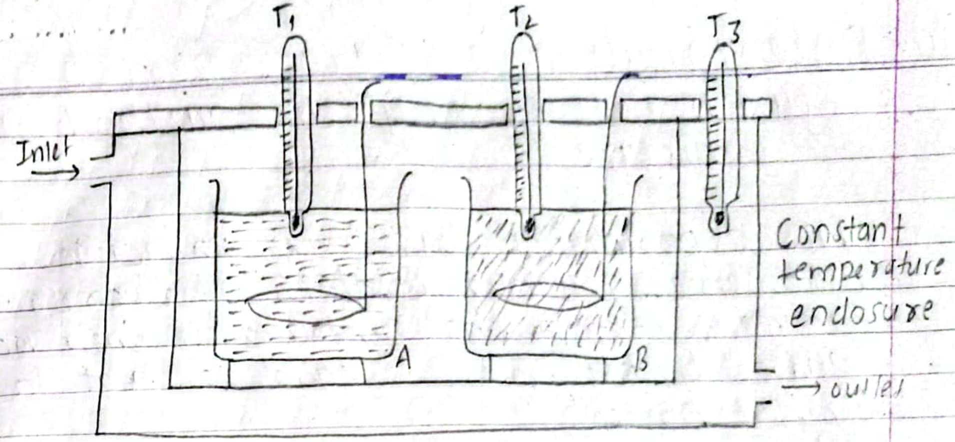


Fig: Experimental arrangent for determination of specific heat capacity of a liquid.

Let, s_w, s_l and s_c be the specific heat capacity of water liquid and caloriemetre respectively. Then,
From Newton's law of cooling,
Rate of cooling of water and caloriemeter A = Rate of cooling of liquid and caloriemeter B

i.e. $\left(\frac{d\theta}{dt}\right)_A = \left(\frac{d\theta}{dt}\right)_B$

or, $\frac{(m_c s_c + m_w s_w)(\theta_2-\theta_1)}{t_1} = \frac{(m_c s_c + m_l s_l)(\theta_2-\theta_1)}{t_2}$

or, $\frac{(m_c s_c + m_w s_w)}{t_1} = \frac{(m_c s_c + m_l s_l)}{t_2}$

or, $\frac{(m_c s_c + m_w s_w)}{m_l}\times\frac{t_2}{t_1} - \frac{m_c s_c}{m_l} = s_l$

or, $\boxed{s_l = \frac{1}{m_l}\left[(m_c s_c + m_w s_w)\frac{t_2}{t_1} - m_c s_c\right]}$

This equation is used to determine specific heat capacity of liquid.

10. Determination of Latent heat of fusion of Ice by the method of mixture.

This experiment is based on principle of calorimetry which states, "if there is no heat lost to the surrounding then heat gained by ~~hot~~ cold body is equal to heat lost by hot body".

At first, a caloriemeter with stirrer of mass 'm_1' is taken and 2/3 part of it is filled with 'm_2' mass of water and initial temperature is noted as 'θ_1'. Then, ice of mass 'm_3' of 0°C is added and the final steady temperature is maintained by stirring the mixture which is 'θ_2'.

Let, 'S_c & S_w' be specific heat capacity of caloriemeter and water respectively and 'L_i' be latent heat of ice.

Then,

From principle of calorimetry,

Heat lost by hot body = heat gained by cold body

i.e. Heat lost by water & caloriemeter = Heat gained by ice to melt + Heat gained by melted ice to reach final steady temperature

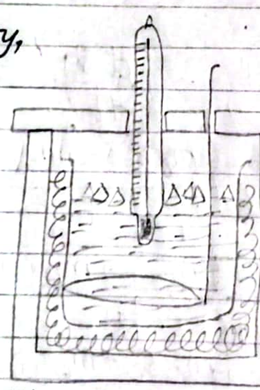


Fig: Measurement of Latent heat of fusion of ice

Or, $(m_1 S_c + m_2 S_w)(\theta_1 - \theta_2) = m_3 L_i + m_3 S_w (\theta_2 - 0)$

or, $L_i = \dfrac{(m_1 S_c + m_2 S_w)(\theta_1 - \theta_2) - m_3 S_w \theta_2}{m_3}$

or, $\boxed{L_i = \dfrac{1}{m_3}[(m_1 S_c + m_2 S_w)(\theta_1 - \theta_2)] - S_w \theta_2}$

This formula is used to determine Latent heat of fusion of ice.

11. Latent heat of steam by the method of mixture.

For the determination of latent heat of steam, a empty caloriemeter with stirrer of mass 'm_1' is taken ~~on~~ of specific heat capacity 'S_c' and 2/3 part of it is filled with water of mass 'm_2' and specific heat capacity 'S_w' such that both will have initial temperature 'θ_1'. Some water is boiled separately in a boiler and steam of mass 'm_3', temperature 'θ_2' and Latent heat of evaporation 'L_v' is passed in the caloriemeter. After that, supply of steam is stopped and final steady temperature of mixture is noted as 'θ_3'.

Then,

From principle of caloriemetry,

Heat lost by hot body = Heat gained by cold body

i.e. Heat lost by steam to evaporate + Heat lost by evaporated steam to reach final steady temperature = Heat gained by water & caloriemeter

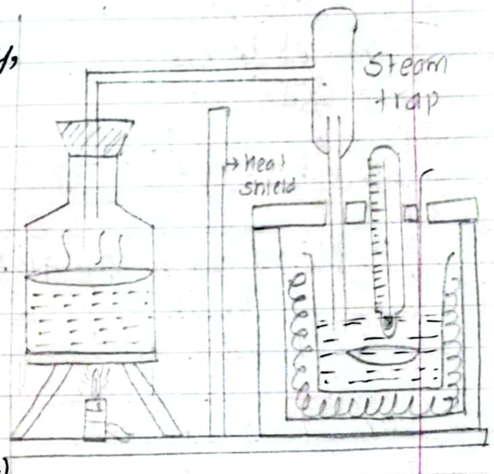


Fig:- Measurement of Latent heat of vaporization of water.

Or, $m_3 L_v + m_3 S_w (\theta_2 - \theta_3) = (m_1 S_c + m_2 S_w)(\theta_3 - \theta_2)$

Or, $L_v = \dfrac{(m_1 S_c + m_2 S_w)(\theta_3 - \theta_1) - m_3 S_w (\theta_2 - \theta_3)}{m_3}$

Or, $\boxed{L_v = \dfrac{1}{m_3}[(m_1 S_c + m_2 S_w)(\theta_3 - \theta_1)] - S_w (\theta_2 - \theta_3)}$

Hence, By knowing parameters on right hand side we can determine Latent heat of vaporization of water.

→ (i) What we need to find
→ (ii) dr. Fig → (iii) Explain (iv) Imp step.

12. Variation of lateral shift with the angle of incidence.

$$d = \frac{t \sin(i-r)}{\cos r}$$

→ Let us consider a glass slab PQRS with thickness t. Let OA be the incident ray on surface RS and AB be the refracted ray on glass slab and BC is emergent ray. Let, 'BM' be the perpendicular distance between direction of incident ray and emergent ray called lateral shift. 'N_1N_2' & 'N_3N_4' are two normals on surface RS and PQ respectively.

In ΔABM,

$$\boxed{\sin(i-r) = \frac{BM}{AB}}$$

$BM = \sin(i-r)\,AB$ —(i)

In ΔABN_2,

$$\boxed{\cos r = \frac{AN_2}{AB}}$$

or, $AB = \dfrac{t}{\cos r}$ —(ii)

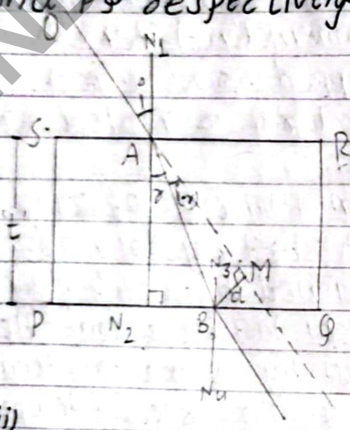


Fig:- Lateral shift

Putting value of AB from (ii) in (i),

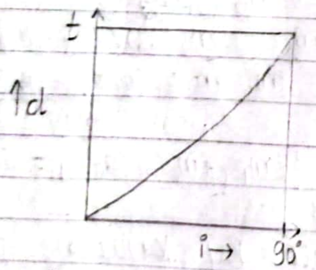
$$BM = \sin(i-r)\frac{t}{\cos r}$$

or, $$\boxed{d = \frac{t\sin(i-r)}{\cos r}}$$

When, $i = 90°$

$$d = \frac{t\sin(90-r)}{\cos r} = \frac{t\cos r}{\cos r}$$

$\therefore d = t$



It means when the angle of incidence is 90° then lateral shift produced by glass will be equal to thickness of slab.

13. Real depth and apparant depth.

$$\mu = \frac{\text{Real depth}}{\text{Apparant depth}}$$

When an object placed at denser medium is viewed from rarer medium then the object appears to be in lesser depth than its real depth due to refraction of light.

→ Let us consider an object 'O' kept in bottom of beaker containing water where XY plane separates air and water medium. Then, the ray OA coming from bottom normally with pass XY without bending along AD. And another ray OB will be refracted away from normal along BC. Then, while viewing from top the ray of light seems to be coming from I which is intersection of OD & BC. And, AI is considered as apparant depth and OA is considered as real depth as in the figure.

Then,

$\boxed{{}_a\mu_w = \dfrac{\sin r}{\sin i}}$ where, i is angle of incidence & r is angle of refraction.

Now, In ΔABI & ΔAOB respectively,

$$\boxed{\sin r = \frac{AB}{BI} \quad \& \quad \sin i = \frac{AB}{OB}}$$

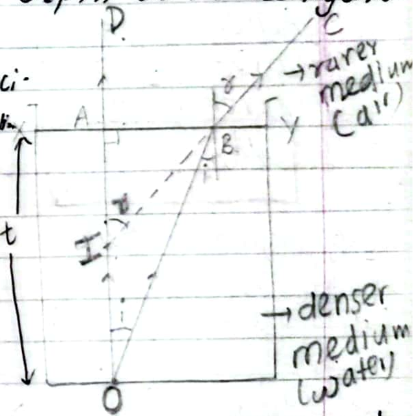


Fig: Real and Apparant depth of object O.

Then,

$$\boxed{{}_a\mu_w = \frac{\frac{AB}{BI}}{\frac{AB}{OB}} = \frac{OB}{BI}}$$

If point B is very close to A then, $OB \approx OA$ & $BI \approx AI$. So,

$$\boxed{{}_a\mu_w = \frac{OB}{BI} = \frac{OA}{AI} = \frac{\text{Real depth}}{\text{Apparant depth}}}$$

Note: Apparant shift $(d) = OI = OA - IA$ = Realdepth − App depth
(When, Realdepth = t) then, $d = t - \dfrac{t}{{}_a\mu_w}$ $\boxed{d = t\left(1 - \dfrac{1}{{}_a\mu_w}\right)}$

14. Total Internal reflection (TIR) and Critical angle.

~~I~~Critical angle :- The angle of incidence in the denser medium for which angle of refraction on rarer medium is 90° is called critical angle of that medium

TIR: If the angle of incidence in denser medium is greater than critical angle then the light ~~inte~~ reflects internally in same medium is called TIR.

→ Let us consider an incident ray PQ passes from denser to rarer medium such that its angle of incidence is equal to critical angle i.e. $\boxed{i=C}$. Then the refracted ray QR will have angle $\boxed{90°}$ with normal.

From snell's law,
when light passes from water to air (or denser to rarer medium)

$$\boxed{{}_w\mu_a = \frac{\sin i}{\sin r}}$$

or, $${}_w\mu_a = \frac{\sin c}{\sin 90°}$$

or, $${}_w\mu_a = \sin c$$

or, $$\frac{1}{{}_a\mu_w} = \sin c$$

$\therefore$ $${}_a\mu_w = \frac{1}{\sin c}$$

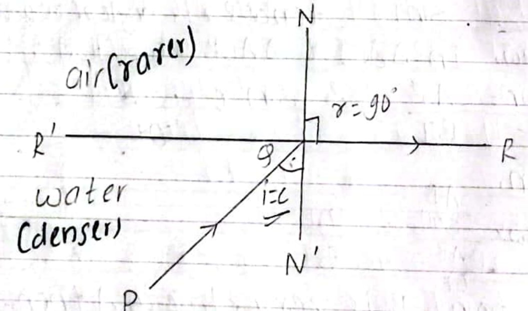


Fig: Critical angle.

This gives the relation between refractive index and critical angle.

15. Measuring refractive index of prism through minimum deviation method.

$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

→ Prism: A prism is a trasparent refracting medium bounded by two plane surfaces inclined to each other at certain angle.

Angle of prism: The angle between two refracting faces of prism is called angle of prism.

~~An~~ Minimum deviation: The minimum value of the angle of deviation when ray of light passes through the prism is called minimum deviation.

→ Let, on a prism XYZ, a ray of light PQ incident on XY surface with angle 'i' refract along QR with angle 'r_1' and by making 'r_2' as angle of incidence in second face it emerges out along RS with angle 'e'. The angle between direction of incident ray and emergent ray is called angle of deviation 'δ'. Let, ∠LQR = α & ∠TRQ = β.

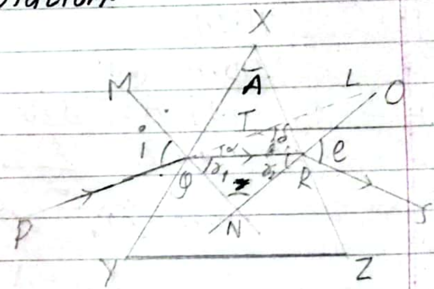


Fig:- Showing refraction of light by prism

From the given figure,

$$i = \alpha + r_1 \quad \text{—(i)}$$

$$e = \beta + r_2 \quad \text{—(ii)}$$

Adding (i) & (ii),

$i+e = \alpha + r_1 + \beta + r_2$

or, $i+e = \delta + r_1 + r_2$ — (iii) [$\because \delta = \alpha + \beta$]

Again,

In Quad XQNR,

$A + 90° + 90° + \angle QNR = 360°$

or $A + \angle QNR = 180°$ — (iv)

Similarly in Δ QNR,

$r_1 + r_2 + \angle QNR = 180°$ — (v)

Comparing (iv) & (v),

$A = r_1 + r_2$ — (vi)

So,

$\boxed{i+e = \delta + A}$ — (vii) [From (vi) & (iii)]

For minimum deviation,

i.e. when $\delta = \delta_m$, $i = e$

$r_1 = r_2 = r$

Then,

From (vi),

$A = r + r$

or, $r = \frac{A}{2}$ — (viii)

From (vii),

$i + i = \delta_m + A$

or, $i = \frac{A + \delta_m}{2}$ — (ix)

From Snell's law,

$_a\mu_g = \frac{\sin i}{\sin r}$

or $\boxed{\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}}$ [From (viii) & (ix)]

Graph on minimum deviation:

*Depends upon:-

(i) Angle of incident on first face

(ii) Angle of prism

(iii) material of prism (μ)

When angle of incidence is increased from zero, angle of deviation first decreases then it becomes lower at some point then increases.

* At minimum deviation:

(i) The light rays passes symmetrical through the prism

(ii) Angle of incident = angle of emergence [$i=e$]

(iii) Refraction angle on first face (r_1) = ~~refraction~~ incident angle on second face (r_2). Then, refracted ray // base of prism (eq.)

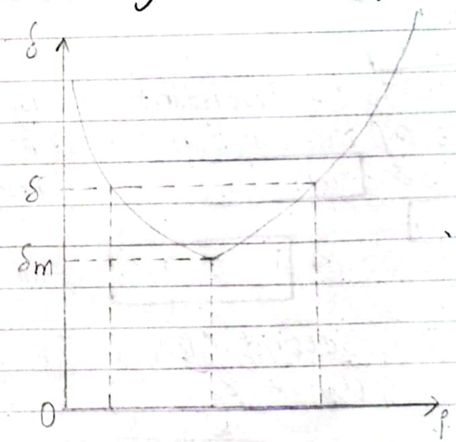


Fig: variation of δ with i in a prism

#Note: There will be two angle of ~~deviati~~ incidence for a angle of deviation. But, for minimum deviation there will be single angle of incidence.

16. Deviation through a small angled prism

[The deviation produced by a small angle prism for small angle of incidence is independent of angle of incidence].

→ Let us consider a small angled prism XYZ. A ray of light PQ incident in XY face with angle 'i' refracts making an angle 'r_1' and it is incident on face XZ as QR making an angle 'r_2' and emerge out along XZ as RS making angle 'e'. Let, 'δ' be the angle of deviation. The angle of deviation is given by,

$\boxed{\delta = i - r_1 + e - r_2}$

$\boxed{\delta = (i+e) - (r_1 + r_2)}$ —(i)

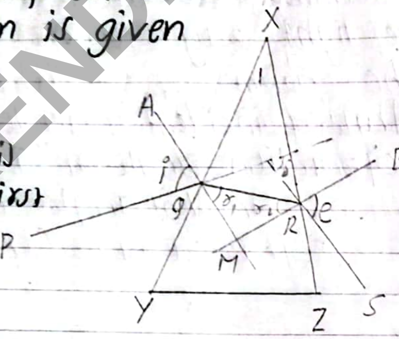


Fig:- Deviation through small angled prism.

Refractive index on first face is given by,

$\boxed{{}_a\mu_g = \frac{\sin i}{\sin r_1}}$

Since, angle of incident and angle of refraction are small, so $\boxed{\sin i \approx i}$ & $\boxed{\sin r_1 \approx r_1}$

${}_a\mu_g = \frac{i}{r_1}$ or, $\boxed{i = r_1 \, {}_a\mu_g}$ —(ii)

Similarly, on second face,

${}_g\mu_a = \frac{r_2}{e}$

$\therefore \boxed{e = r_2 \, {}_a\mu_g}$ —(iii)

From eq^n (i), (ii) & (iii),

$\delta = r_1 \mu + r_2 \mu - (r_1 + r_2)$

or, $\delta = \mu(r_1 + r_2) - (r_1 + r_2)$

$\therefore \boxed{\delta = A(\mu - 1)}$ $[\because r_1 + r_2 = A]$

Chapter: 3
Kinematics

17. Projectile fired with an angle with horizontal.

Let, the projectile is fired with initial velocity 'u' from point O making an angle 'θ' with the horizontal. Then u can be resolved into two parts (i) $v_x = u\cos\theta$ which is constant throughout the motion and (ii) $v_y = u\sin\theta$ vertically upward which changes due to acceleration due to gravity (g). Let, P(x,y) be the point of projectile at any instant of time 't'. Then,

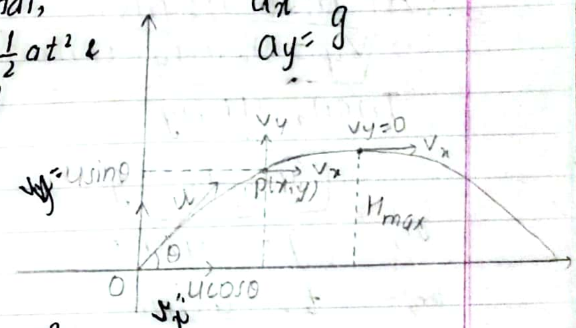
$a_x = 0$
$a_y = g$

Motion along horizontal,

$x = v_x t$ $[\because s = ut + \frac{1}{2}at^2$ & $a_x = 0]$

or, $x = u\cos\theta \, t$

or, $\boxed{t = \frac{x}{u\cos\theta}}$ —(1)



Motion along vertical,

$\boxed{y = v_y t - \frac{1}{2} g t^2}$

or, $y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \times \frac{x^2}{u^2\cos^2\theta}$

or, $\boxed{y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}} x^2$ —(ii)

Above equation is the equation of parabola. So, we see that the path followed by the projectile is parabolic.

Maximum height (H_{max}):-

It is the greatest height to which the projectile rises from the ground.

At maximum height, $\boxed{v_y = 0}$

$S_y = H$, ~~$v_y = 0$~~, ~~$v_x = u\cos\theta$~~ $\boxed{v_y = u\sin\theta}$,

$\boxed{a_y = -g}$

The from equation,

$(v_y)^2 = (u_y)^2 - 2gs_y$

or, $0 = u^2\sin^2\theta - 2gH$

or, $H = \dfrac{u^2\sin^2\theta}{2g}$

Time of ascent:

It is the time taken by the projectile to attain the maximum height.

At max. height,

$\boxed{v_y = 0}$, $\boxed{u_y = u\sin\theta}$, $\boxed{a_y = -g}$

Then, using

$\boxed{v_y = u_y - gt}$

or, $gt = u\sin\theta$

or, $t = \dfrac{u\sin\theta}{g}$

Time of flight (T)

$T = 2t = \dfrac{2u\sin\theta}{g}$

∵ Time taken by projectile to reach maximum height = time taken to reach the ground from the maximum height.

Horizontal range (R)

It is the horizontal distance covered by projectile.

R = Horizontal velocity × Time of flight

$= u\cos\theta \times \dfrac{2u\sin\theta}{g}$

$\therefore R = \dfrac{u^2\sin 2\theta}{g}$

Mechanics:-

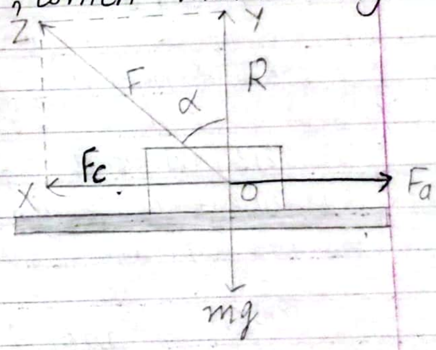
Chapter-4

Laws Of Motion

18. Relationship between angle of friction and angle of repose.

Angle of friction is the angle made by resultant of limiting friction and normal reaction with the normal reaction.

Let, a force F_a is applied to a block ~~resting~~ kept over a table so that block just begins to move which is equal to the limiting friction F_c and normal reaction $R = mg$. So that resultant of F_c and R is F, which makes angle α with R.



From figure,

In ΔOZY,

$\tan\alpha = \dfrac{ZY}{OY}$

or, $\tan\alpha = \dfrac{F_c}{R}$ $[\because YZ = F_c \; \& \; OY = R]$

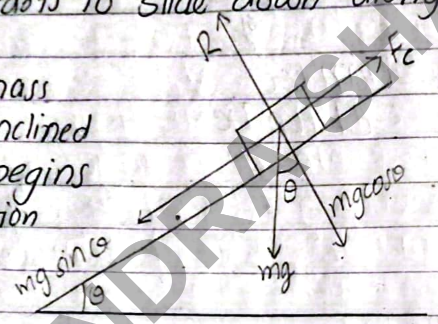
or, ~~$F_c = R\tan$~~ As, $F_c = \mu R$

$\therefore \tan\alpha = \mu$

Hence, coefficient of limiting friction is equal to tangent of angle of friction.

Angle of repose is defined as the minimum angle of inclination of plane structure with horizontal such that a body kept on plane just starts to slide down along the plane.

Let a body of mass 'm' is kept on a inclined plane such it just begins to slide at inclination 'θ' with horizontal.



The component of weight mg normal to plane mgcosθ balances normal reaction 'R' and the component which is along inclined plane mgsinθ balance frictional force acting upward along the surface.

In equilibrium,

$$mg\sin\theta = F_c \quad -(i)$$

$$mg\cos\theta = R \quad -(ii)$$

Dividing (i) by (ii),

$$\tan\theta = \frac{F_c}{R}$$

or, $\tan\theta = \mu \quad [\because \mu = \frac{F_c}{R}]$

∴ The tangent of angle of repose is equal to coefficient of friction between the surface.

As we know, $\tan\alpha = \mu$ where α is angle of friction.

Then, $\tan\theta = \tan\alpha$

$\therefore \theta = \alpha$

∴ Angle of repose is equal to angle of friction.

19. Principle of conservation of linear momentum:

It states that, "In the absence of external forces, the total linear momentum of the system remains conserved."

Let two bodies of masses m_1 and m_2 be moving in a straight line with initial velocity 'u_1' and 'u_2' respectively. If $u_1 > u_2$ they collide after some time. Let, t be the time of collision. After that they move with final velocity of 'v_1' and 'v_2' respectively.

So,

Force experienced by body A due to B (F_{AB})

$$= \frac{\text{change in momentum}}{\text{time}}$$

$$= \frac{m_1v_1 - m_1u_1}{t}$$

Similarly, Force experienced by B due to A (F_{BA})

$$= \frac{\text{change in momentum}}{\text{time}} = \frac{m_2v_2 - m_2u_2}{t}$$

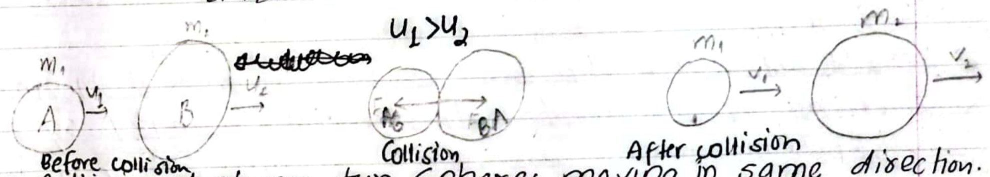


Fig: Collision between two spheres moving in same direction.

From Newton's third law,

$$F_{AB} = -F_{BA}$$

Or, $\frac{m_1v_1 - m_1u_1}{t} = -\frac{(m_2v_2 - m_2u_2)}{t}$

Or, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

∴ Total momentum before collision = Total momentum after collision

Thus, principle of conservation of linear momentum is verified.

Newton's second law: Real law of motion

Both first and third law are contained in second law. Thus, second law is called as real law of motion.

According to newton's 2nd law, the net external force applied on a body is equal to rate of change of linear momentum.

i.e. $F = \frac{dp}{dt}$

If net external force is 0, $\boxed{F_{ex} = 0}$

$dp = 0$

Integrating the equation, we get

$mv = $ constant

or, $v = \frac{\text{contant}}{\text{mass}} = $ constant

Thus, in absence of net external force body ~~at~~ moving at uniform velocity continues to move with same velocity and body at rest remains at rest forever. which is Newton's first law of motion.

Similarly,

Let us consider two bodies A and B moving along a straight path in an isolated system. Let, two bodies collide with each other for time 't'. Then as a result of collison their velocity changes along with linear momentum.

Change in linear momentum of A,

$\Delta P_A = \vec{F}_{AB} \times t$

Change in linear momentum of B,

$\Delta P_B = \vec{F}_{BA} \times t$

Total momentum $(\Delta P) = F_{AB} \times t + F_{BA} \times t$

If no external force act on a system,

$\Delta P = 0$

or, $F_{AB} \times t + F_{BA} \times t = 0$

$\therefore F_{AB} = -F_{BA}$

which is Netwon's third law of motion.

Newton's first law of motion:-

It states that, "Every body in this universe continues in its initial state of rest or uniform motion in a straight line unless any external force is acted on a body to change its state".

Newton's Third law of motion states that,

"To every action there is equal and opposite reaction."

Chapter- 5
Work, Energy and Power

20. **Energy conservation for a freely falling Body under the effect of gravity.**

→ Principle of conservation of energy states that, "Energy can neither be created nor be destroyed but can be transformed from one form to another so that the total energy of the system always remains constant or conserved."

Let, a body of mass m at rest is at the height h above the ground level at point A.

Then, at point A

K.E. of the body $= 0$

P.E. of the body $= mgh$

$\therefore$ Total energy of the body at point A $=$ K.E $+$ P.E.

$= mgh$

Let, the body falls freely from A to the ground and at a point B on its path, its height above the ground will be $(h-x)$. Then,

P.E. of the body $= mg(h-x)$

If v_B is the velocity at B, then

$v_B^2 = u^2 + 2gs$

or, $v_B^2 = 2gx$

$\therefore$ K.E. of the body $= \frac{1}{2} m v_B^2 = \frac{1}{2} m \times 2gx$

$= mgx$

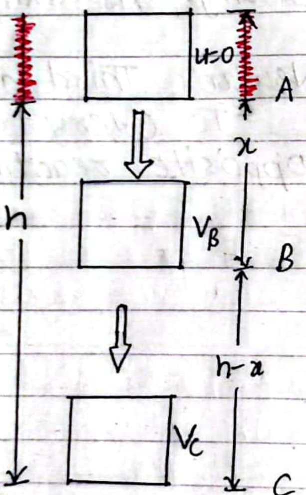


Fig:- Body falling under the effect of gravity

$\therefore$ Total energy of the body $=$ K.E. $+$ P.E. $=$

$= mgx + mg(h-x)$

$= mgh$

At point C i.e. at ground level,

If v_C is velocity of body at point C (just before striking the ground), then

$v_C^2 = 0 + 2gh$

$v_C^2 = 2gh$

$\therefore$ K.E. of body $= \frac{1}{2} m v_C^2 = \frac{1}{2} m \times 2gh = mgh$

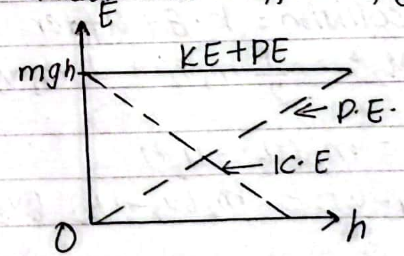
P.E. of body $= mgh = 0$

$\therefore$ Total energy of the body $=$ K.E. $+$ P.E.

$= mgh + 0 = mgh$

Thus, total mechanical energy of the body remains same throughout the journey when the body is falling under the effect of gravity. At maximum height, potential energy is equal to mgh and kinetic energy is 0 and as the body falls downward potential energy decreases and kinetic energy increases such that total mechanical energy is mgh. Finally at ground, P.E is 0 and K.E. is equal to mgh.

Graph showing variation of K.E. and P.E. of a body falling under the effect of gravity:-



21. Elastic collision in one dimension

If the kinetic energy and momentum are conserved in the collision, it is said to be elastic collision.

In elastic collision:-

(i) The momentum is conserved.
(ii) Kinetic energy is conserved.
(iii) Total energy is conserved.
(iv) Forces involved are of conservative nature.

Let us consider two objects of masses m_1 and m_2 moving with velocities u_1 and u_2 such that $u_1 > u_2$ in same path and let ~~eth~~ after collision their velocities be v_1 and v_2 on the same line.

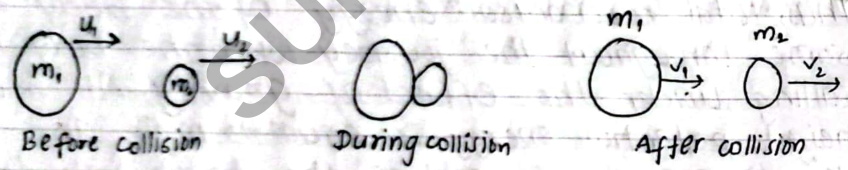


Fig:- Elastic collision between two bodies.

From principle of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad -(i)$$

or, $m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad -(ii)$

Since collision is elastic,

K.E. before collision = K.E. after collision

or, $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

or, $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

or, $m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 - u_2)(v_2 + u_2) \quad -(iii)$

Dividing eqn (iii) by eqn (ii),

$u_1 + v_1 = u_2 + v_2$

or, $u_1 - u_2 = v_2 - v_1 \quad -(iv)$

$v_2 = u_1 - u_2 + v_1 \quad -(v)$

From eqn (i) & (v)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2[u_1 - u_2 + v_1]$

~~or, $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$~~

~~or, $m_1(u_1 - v_1) = (m_1 + m_2)v_1 + m_2u_1 - 2m_2u_2$~~

or, $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$

or, $(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

or, $v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2} \quad -(vi)$

Also, from eqn (v),

$v_1 = u_2 - u_1 + v_2 \quad -(vii)$

Substituting value of v_1 in eqn (i),

$m_1u_1 + m_2u_2 = m_1(u_2 - u_1 + v_2) + m_2v_2$

or, $m_1u_1 + m_2u_2 = m_1u_2 - m_1u_1 + m_1v_2 + m_2v_2$

or, $2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2$

or, $v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2} \quad -(viii)$

Special cases,

(i) When $m_1 = m_2$,
$v_1 = u_2$ and $v_2 = u_1$
i.e. two bodies will exchange their velocities.

(ii) When $u_2 = 0$ i.e. when 2nd body is at rest. Then,
$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1$ & $v_2 = \frac{2m_1u_1}{m_1 + m_2}$

(iii) When, $m_2 >> m_1$ and $u_2 = 0$
$v_1 \approx -u_1$ and $v_2 \approx 0$
i.e. lighter body will have reverse velocity and heavy body will be at rest.

(iii) When, $m_1 >> m_2$ and $u_2 = 0$
$v_1 \approx u_1$
& $v_2 \approx 2u_1$
i.e. lighter body velocity ~~will be same and heavier body~~ will be double of initial velocity of massive and heavy

22. Inelastic collision in one dimension:

The collision in which momentum is conserved but K.E. is not conserved is called inelastic collision. In it, particles stick permanently together on impact.

In inelastic collision:

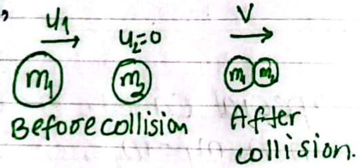
(i) Momentum is conserved.
(ii) Total energy is conserved.
(iii) Kinetic energy is not conserved.
(iv) Forces involved are of non-conservative in nature.
(v) Mechanical energy is converted to other forms.

→ Let us consider two perfectly inelastic bodies A and B of mass m_1 and m_2 has velocity u_1 and is at rest respectively. After some time they will collide and move together with common velocity v. Then,

~~Initial~~ momentum before collision $= m_1 u_1$

Momentum after collision $= (m_1+m_2)v$

Since, momentum is conserved,



$$m_1 u_1 = (m_1+m_2)v$$

or $$v = \frac{m_1 u_1}{m_1+m_2} \quad \text{—(i)}$$

Now,

$$\frac{\text{K.E. before collision}}{\text{K.E. after collision}} = \frac{\frac{1}{2} m_1 u_1^2}{\frac{1}{2}(m_1+m_2)v^2}$$

$$= \frac{m_1 u_1^2}{(m_1+m_2)\frac{m_1^2 u_1^2}{(m_1+m_2)^2}}$$

$$= \frac{m_1+m_2}{m_1} > 1$$

∴ K.E before collision > K.E. after collision

23. Work done by a Variable forces

Let, us consider a body is displaced from A to B by variable force. Let, the entire displacement of a body from A to B is made up of infinitesimal displacements. Let one such displacement be PQ i.e. PQ = dx. Since, PQ is very small displacement we consider force to be constant in magnitude and direction. Thus, work done for that displacement is given by:-

$$dW = F dx$$
$$= PS \times PQ$$
$$= \text{Area of strip PQRS}$$

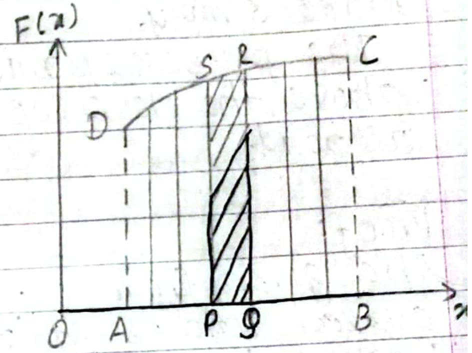


Fig: Work done by variable force.

Then, total work done in this interval i.e. from A to B is given by

$$W = \Sigma dW$$
$$= \Sigma F dx$$

If the width of the strip is reduced so that it approaches zero i.e., $dx \to 0$, Then we can write,

$$W = \lim_{dx \to 0} \Sigma F dx$$

If the force is continuous force then, the summation sign is replaced by integration sign as:

$$W = \int_{x_A}^{x_B} F\, dx$$

$$= \int_{x_A}^{x_B} \text{Area of strip PQRS} = \text{Area of ABCDA}$$

Hence, work done by variable force is numerically equal to area under the force curve on the displacement axis.

Chapter-6

Circular motion

24. Expression for the banking angle:

Banking of Road: In hilly regions, the vehicle has to take turn around a curve path with reasonable speed, a greater centripetal force is required which can be managed by making an inclined road such that the outer end of the road higher than inner road which is called as banking of a curved path.

More simply,
The process of raising outer edge of curved road above the level of inner road is called banking of road.

Proof:-
The given figure shows the curved road AB with radius r banked at an angle θ. Suppose the car goes ~~around this~~ around this curve with a speed v. The various force acting on car are:
(i) The weight mg of the car acting vertically downward.
(ii) The normal reaction R acting normal to the road AB.

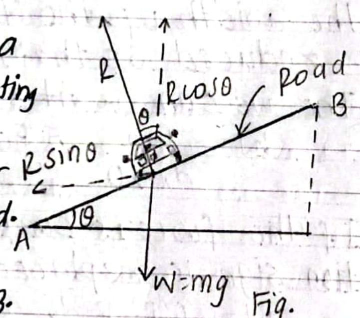


Fig.

R is at an angle θ with vertical such that its horizontal component is $R\sin\theta$ and vertical component $R\cos\theta$.
The vertical component balances weight 'mg' of car
i.e. $R\cos\theta = mg$ —(i)
Similarly,

The horizontal component $R\sin\theta$ provides necessary centripetal force to the car.
i.e. $R\sin\theta = \frac{mv^2}{r}$ —(ii)

Dividing (ii) by (i),
$$\boxed{\tan\theta = \frac{v^2}{rg}} \quad \text{—(iii)}$$

Eqn (iii) gives the banking angle θ for curved path.
Note:- For the given radius, no one angle is correct for all speeds.

25. Conical Pendulum:-

A conical pendulum is a very small mass suspended by string from rigid point and whirled round the horizontal circle with constant speed. Let us consider a bob of mass M tied to a string of length l is whirled in a horizontal circle of radius r with velocity v such that string makes angle θ with vertical. When bob is at point A, the tension F in the string has two components $F\sin\theta$ and $F\cos\theta$. $F\cos\theta$ balances weight of bob and $F\sin\theta$ provides necessary centripetal force.

∴ $F\cos\theta = mg$ —(i)
$F\sin\theta = \frac{mv^2}{r}$ —(ii)

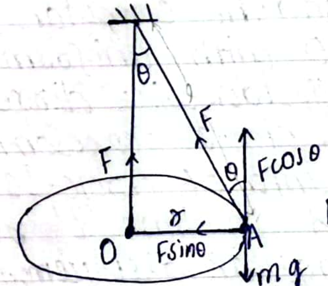


Fig: Conical pendulum

Dividing (ii) by (i),
$\tan\theta = \frac{v^2}{rg}$
or $v^2 = \tan\theta \, rg$
or $g\tan\theta = \frac{v^2}{r}$

if ω is the angular speed of the bob then, $v = r\omega$
∴ $\tan\theta = \frac{r^2\omega^2}{rg}$

or $\omega = \sqrt{\frac{g\tan\theta}{r}}$

Time period (T) of conical pendulum is given by.

$T = \frac{2\pi}{\omega}$

or, $\frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{r}}$

or $T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

or $T = 2\pi\sqrt{\frac{l\sin\theta}{\frac{g\sin\theta}{\cos\theta}}} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$\therefore \boxed{T = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$

This gives time period of conical pendulum.

26. Expression for Centripetal acceleration

$a = \frac{v^2}{r} = r\omega^2$

Let us consider a body of mass 'm' is moving along the circular path of radius 'r' and centre at O with uniform angular velocity 'ω' in anti-clockwise direction.

Let, circle represented by x-y plane whose origin is center of circle. Let, the particle starts from A and in time 't' reaches point P(x,y) with angular displacement $\angle POA = \boxed{\theta = \omega t}$ $[\because \omega = \frac{\theta}{t}]$

Then,

the position vector $\vec{r}$ of a particle at time t is given by $\boxed{\vec{r} = x\hat{i} + y\hat{j}}$

$= r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

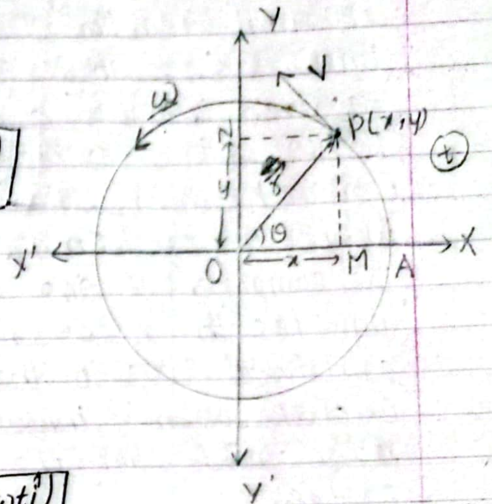
$\omega = \frac{\theta}{t}$

$= r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$\boxed{\vec{r} = r(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})}$ —(1)

And, velocity of the particle at time t is given by:-

[Note: Here, $\vec{v}$ refers centripetal velocity]



$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{r\,d(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})}{dt}}$

[From (1)]

$= r[-\omega\sin\omega t\,\hat{i} + \omega\cos\omega t\,\hat{j}]$

$= r\omega[\cos\omega t\,\hat{j} - \sin\omega t\,\hat{i}]$

$\therefore \vec{v} = -r\omega[\sin\omega t\,\hat{i} - \cos\omega t\,\hat{j}]$

Acceleration of the particle at the time t is given by:-

$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = -r\omega\frac{d}{dt}(\sin\omega t\,\hat{i} - \cos\omega t\,\hat{j})}$

$= -r\omega(\omega\cos\omega t\,\hat{i} + \omega\sin\omega t\,\hat{j})$

$= -r\omega^2[\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}]$

$\boxed{\vec{a} = -\omega^2\vec{r}}$ [From (1)]

Negative sign shows that direction of acceleration($\vec{a}$) is opposite to the direction of $\vec{r}$ (O to A) i.e. the acceleration is directed along the radius towards the center of circular path. Hence, magnitude of centripetal acceleration is given by

$\boxed{a = r\omega^2}$

or $a = r\left(\frac{v}{r}\right)^2$ $[\because v = \omega r]$

$\boxed{\therefore a = \frac{v^2}{r}}$

27. Bending of Cyclist $\left(\tan\theta = \frac{v^2}{rg}\right)$

Let, us consider a cyclist of mass m, moving with the constant velocity 'v' in the circular road of radius 'r' tilting angle θ with the vertical. The normal reaction R of the ground acts along the line making angle θ with vertical. The R can be resolved into two components, $R\cos\theta$ acting vertically upward which balances weight of cyclist mg and the component $R\sin\theta$ provides the necessary centripetal force to the cyclist which is horizontal to the road along the centre.

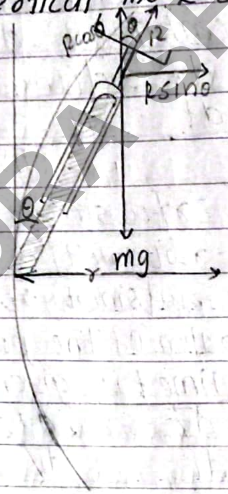


Fig:- Bending of cyclist in a circular path.

Then,

$$R\cos\theta = mg \quad \text{—(i)}$$

$$R\sin\theta = \frac{mv^2}{r} \quad \text{—(ii)}$$

Dividing (ii) by (i) we get

$$\tan\theta = \frac{v^2}{rg}$$

This relation gives the required angle through which cyclist tilts with vertical.

Chapter-9

Rotational Dynamics

28. Calculation of Moment of Inertia of Rigid bodies (Thin uniform Rod)

(a) About the axis passing through the centre and perpendicular to its length.

→ Let us consider a thin uniform rod of length 'l', mass 'M' rotating about an axis YOY' passing through its centre. Let, we consider a small elemental mass 'dm' of length 'dx' at a distance 'x' from the centre O. Then,

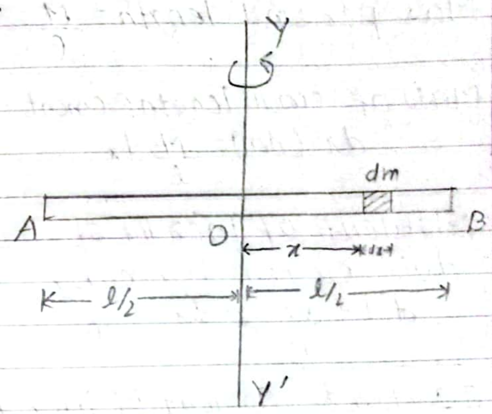


Fig: Moment of inertia of thin uniform rod about an axis passing through its centre.

Mass per unit length of rod $= \frac{M}{l}$

Mass of small element of length dx, $dm = \frac{M}{l}dx$

Moment of inertia of a ~~unit~~ elemental mass is given as,

$$dI = \frac{M}{l}dx \times x^2 \quad \text{—(1)}$$

Then total moment of inertia of the thin uniform rod AB can be calculated by integrating eq^n (1) from $x = -l/2$ to $x = l/2$

Then,

$$I = \int dI = \int_{-l/2}^{l/2} \frac{M}{l} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$= \frac{M}{l}\left[\frac{l^3}{8\times3} + \frac{l^3}{8\times3}\right]$$

$$\therefore I = \frac{Ml^2}{12}.$$

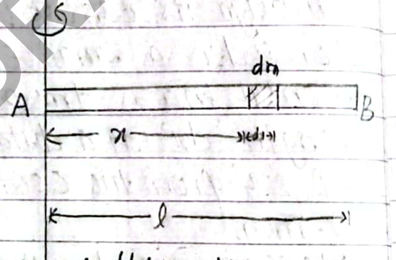
Similarly,

(b) Moment of inertia of thin uniform rod about an axis passing through one end and perpendicular to length.

→ Let us consider a thin uniform rod AB of length 'l' and mass 'M' rotating about an axis passing through end A and perpendicular to its length. Consider a elemental mass (dm) of length (dx) at a distance of x from the axis of rotation. Then

Mass per unit length $= \frac{M}{l}$

Mass of small length element dx $(dm) = \frac{M}{l} dx$



Moment of inertia of this element is given by

$$dI = \frac{M}{l} x^2 dx \quad \text{---(1)}$$

Fig: MI of thin uniform rod about an axis passing through its one end and perp. to its length.

To find moment of inertia of whole rod AB we should integrate eqn (1) from limits $x=0$ to $x=l$, then we have

$$I = \int dI = \int_0^l \frac{M}{l} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l$$

$$\therefore I = \frac{Ml^2}{3}$$

Note:

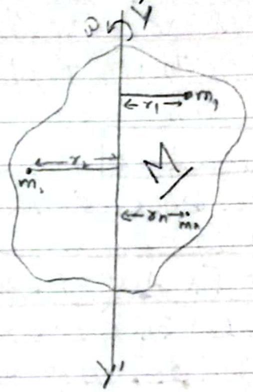
* Moment of inertia of a rigid body about an axis of rotation is the sum of the product of individual mass with the respective squares of the distances from the axis of rotation.
* Radius of Gyration is defined as the perpendicular distance from the axis of rotation to a point where the entire mass of the body is supposed to be concentrated such that the MI remains same about an axis of rotation.

29. Relation between torque, Moment of inertia and angular acceleration.

$$\boxed{\tau = I\alpha}$$

→ Let us consider a rigid body having mass M and rotating about the axis of rotation YY' with angular velocity ω. Suppose the body consist of small n particles of masses $m_1, m_2 \ldots m_n$ ~~and~~ at the perpendicular distance from YY' $r_1, r_2 \ldots r_n$ respectively.

Let, τ be the external torque which produces constant angular acceleration on all particles but different linear acceleration of n particles as $a_1 = r_1\alpha$, $a_2 = r_2\alpha \ldots a_n = r_n\alpha$ $[\because a = r\alpha]$ respectively.



The Force on n particles is given by $F_1 = m_1 a_1 = m_1 r_1 \alpha$, $F_2 = m_2 a_2 = m_2 r_2 \alpha \ldots$ $F_n = m_n a_n = m_n r_n \alpha$ and torque about axis of rotation is given as $\tau_1 = r_1 F_1 = m_1 r_1^2 \alpha$, $\tau_2 = r_2 F_2 = m_2 r_2^2 \alpha \ldots \tau_n = r_n F_n = m_n r_n^2 \alpha$

Thus, The total torque of a rigid body is sum of individual torques of n particles.

$$\therefore \text{Total torque}(\tau) = \tau_1 + \tau_2 + \tau_3 + \cdots \tau_n$$
$$= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \cdots + m_n r_n^2 \alpha$$
$$= (m_1 r_1^2 + m_2 r_2^2 + \cdots m_n r_n^2)\alpha$$
$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

$\therefore \boxed{\tau = I\alpha}$ [where, $I = \sum_{i=1}^{n} m_i r_i^2$ is the moment of inertia about YY'

Further, when $\alpha = 1$, $\tau = I$. So,

Moment of inertia of body about axis of rotation is equal to the torque required to produce unit angular acceleration of body about that axis.

30. Work done by couple & Power in rotational motion.

Couple $\boxed{W = \tau \times \theta}$ $\boxed{P = \tau\omega}$

Couple:- Two equal and unlike parallel forces acting on two different points of rigid body is called couple. Torque due to couple is given by τ = magnitude of either force × ⊥r distance between the forces

→ Let us consider two equal and unlike parallel forces each of magnitude F acting on A and B. Let AB be the diameter of wheel. Let, the wheel rotates through the angle θ such the point A, B shifts to A', B'. Then Torque due to couple is given by

$\boxed{\tau = F \times 2r}$

Then, Work done by couple of forces is

$\boxed{W = F \times AA' + F \times BB'}$

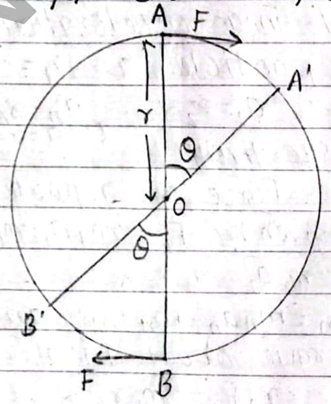
$= F \times r\theta + F \times r\theta$

[∵ For small θ, AA' = BB' = s & $\theta = \frac{s}{r} \Rightarrow s = r\theta$]

$= 2Fr\theta$

$= F\theta \times 2r = \tau\theta$ [∵ $\tau = F2r$]

∴ Work done by couple $(W) = \tau\theta$ —①



→ Differentiating eqⁿ ① w.r.t. t,

$\frac{dW}{dt} = \frac{d\tau\theta}{dt}$

or $P = \tau \frac{d\theta}{dt}$

or $\boxed{P = \tau\omega}$

i.e. Power in rotational motion = torque × angular velocity.

31. Relation between Angular momentum, moment of inertia and angular velocity of rigid body.

$\boxed{L = I\omega}$

Let us consider a rigid body rotating with angular velocity ω about an axis which is perpendicular to the plane of paper. Consider a particles $m_1, m_2, \ldots m_n$ at a perpendicular distance $r_1, r_2, \ldots r_n$ from the axis of rotation. Since, the body is rigid all particle will have same angular velocity (ω). If $v_1, v_2 \ldots v_n$ is the linear velocity, ~~and~~ $p_1, p_2 \ldots p_n$ be linear momentum and $L_1, L_2 \ldots L_n$ be angular momentum of body particles respectively. Then,

$p_1 = m_1 v_1 = m_1 r_1 \omega,\ p_2 = m_2 v_2 = m_2 r_2 \omega \ldots p_n = m_n v_n = m_n r_n \omega$

Again, By definition of angular momentum i.e. $L = rp$,

$L_1 = r_1 p_1 = m_1 r_1^2 \omega,\ L_2 = r_2 p_2 = m_2 r_2^2 \omega \ldots L_n = r_n p_n = m_n r_n^2 \omega$

So,

Total linear momentum (L)

$= L_1 + L_2 + \ldots + L_n$

$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + \ldots + m_n r_n^2 \omega$

$= (m_1 r_1^2 + m_2 r_2^2 + \ldots m_n r_n^2)\omega$

$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$

∴ $L = I\omega$ [∵ $I = \sum_{i=1}^{n} m_i r_i^2$ where I is moment of inertia of body about the given axis]

Fig: A rigid body rotating about an axis YY'.

∴ angular momentum is defined as the product of moment of inertia about an axis of rotation and angular velocity about the same axis.

For rolling body (K.E): $\frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$
$= \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2 = \frac{1}{2}MV^2\left(\frac{K^2}{R^2}+1\right)$

32. Relation between moment of inertia, Rotational kinetic energy & angular velocity.

$$\boxed{K.E_{(rot)} = \frac{1}{2}I\omega^2}$$

→ Let ~~a~~ us consider a rigid body of mass 'M' rotating about axis 'YY'' with constant angular velocity ω. Suppose the body consist of n no. of particles having masses $m_1, m_2 \ldots m_n$ at the perpendicular distance of $r_1, r_2 \ldots r_n$ from axis of rotation. Each particle will have different linear velocity $v_1, v_2 \ldots v_n$ respectively. The linear velocity of particle is given by:-

$v_1 = r_1\omega$, $v_2 = r_2\omega$ ...
$v_n = r_n\omega$. Then rotating K.E of n particles is,

$K.E_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1r_1^2\omega^2$

$K.E_2 = \frac{1}{2}m_2v_2^2 \ldots K.E_n = \frac{1}{2}m_nv_n^2$
$= \frac{1}{2}m_2r_2^2\omega^2 \ldots = \frac{1}{2}m_nr_n^2\omega^2$

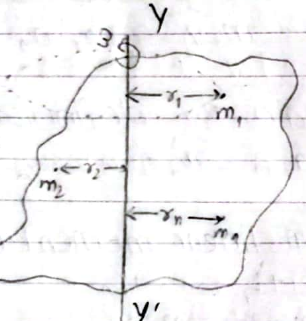


Fig: Rotating body about YY'.

Then total rotational kinetic energy of body is given by: $= K.E_1 + K.E_2 + \ldots K.E_n$

$$K.E_{(rot)} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \ldots \frac{1}{2}m_nr_n^2\omega^2$$

$$= \frac{1}{2}\omega^2(m_1r_1^2 + m_2r_2^2 + \ldots m_nr_n^2)$$

$$= \frac{1}{2}\omega^2\left(\sum_{i=1}^{n} m_ir_i^2\right) = \frac{1}{2}I\omega^2$$ (where $I = \sum_{i=1}^{n} m_ir_i^2$ is moment of inertia about YY')

$$\boxed{\therefore K.E_{(rot)} = \frac{1}{2}I\omega^2}$$

$mK^2 = I$

33. Acceleration of rigid body rolling down an inclined plane.

$$a = \frac{g\sin\theta}{\left(\frac{K^2}{R^2}+1\right)} = \frac{Mg\sin\theta}{\left(M+\frac{I}{R^2}\right)} = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)}$$

→ Let us consider a spherical disc of mass m and radius R, rolling down along a inclined plane with an angle θ with horizontal. If v is the linear velocity acquired by body on moving a distance of s along the plane, such that it descends through a vertical height h and loses potential energy.

Potential energy lost by body = mgh

The loss of potential energy must be equal to the gain in kinetic energy.

$$\therefore \boxed{\text{Total K.E gained by the body} = \frac{1}{2}mv^2\left(\frac{K^2}{R^2}+1\right)}$$

As the disc doesn't slip i.e. (no frictional force), so total mechanical energy is conserved. So,

loss of P.E. = Gain of K.E.

or, $mgh = \frac{1}{2}mv^2\left(\frac{K^2}{R^2}+1\right)$

or, $$\boxed{v^2 = \frac{2gh}{\left(\frac{K^2}{R^2}+1\right)}}$$

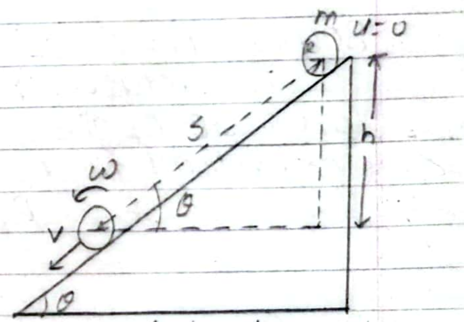


Fig: A body rolling in an inclined plane

From figure,
$h = s\sin\theta$

And, as $u = 0$ $v^2 = 0 + 2as = 2as$

Thus,

or $$\boxed{2as = \frac{2gs\sin\theta}{\left(\frac{K^2}{R^2}+1\right)}} \Rightarrow \boxed{a = \frac{g\sin\theta}{\left(\frac{K^2}{R^2}+1\right)} = \frac{m g\sin\theta}{\left(\frac{mK^2}{R^2}+m\right)} = \frac{mg\sin\theta}{\left(m+\frac{I}{R^2}\right)}}$$

which is the required equation.

Unit-2 → Heat and Thermodynamics.

3. Thermal Properties of Matter

34. Ideal gas equation

$$\boxed{PV = nRT = mrT}$$

Charle's law of pressure:

It states that when volume is kept constant then pressure of given mass of gas is directly proportional to its absolute temperature. i.e.

$$P \propto T.$$

Charle's law of Volume:

It states that at constant pressure, volume of given mass of gas is directly proportional to its absolute temperature.

i.e.

$$V \propto T$$

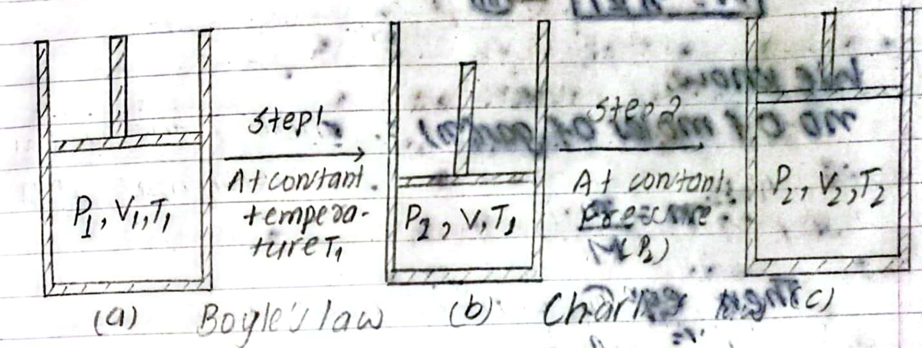
Boyle's law

It states that at constant temperature pressure of given mass of gas is inversely proportional to its volume.

i.e $$P \propto \frac{1}{V}$$

Proof:-

Let us consider 1 mole of gas filled in a non-conducting cylinder fitted with frictionless movable pulley. Let, P_1, V_1 and T_1 be initial Pressure, Volume and Temperature of given mass of gas. Let, this gas is allowed to expand isothermally (T_1 constant) so that its pressure and volume changes to P_2 and V. Again the gas is allowed to expand at constant pressure P_2 such that its temperature and volume becomes T_2 and V_2 respectively.



At first step,
From Boyle's law

$$P_1 V_1 = P_2 V$$

or, $$V = \frac{P_1 V_1}{P_2} \quad \text{—(i)}$$

At second step,
From Charles law of volume.

$$\frac{V}{T_1} = \frac{V_2}{T_2}$$

or $$V = \frac{V_2 \times T_1}{T_2} \quad \text{—(ii)}$$

Combining (i) & (ii).

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This equation is called combined gas equation.

$\frac{PV}{T} = R$, where R is universal gas constant which value is 8.314 J/mol K

or, $PV = RT$ for 1 mole of gas

Then for 'n' mole of gas,

$$\boxed{PV = nRT} \quad \text{—(iii)}$$

We know,

no of moles of gas (n) = $\frac{\text{mass of the gas (m)}}{\text{Molar mass (M)}}$

$\therefore n = \frac{m}{M}$

Then, eqn (iii) can be written as,

$PV = \frac{m}{M} RT$

or $\boxed{PV = m r T}$ where, $r = \frac{R}{M}$ is called the gas constant per unit volume.

35. Expression for the pressure exerted by the ideal gas on the wall of container from postulates of kinetic theory of gas.

$$\boxed{\bar{P} = \frac{1}{3} \rho \bar{C}^2}$$

Let us consider a cubical container of length 'l' have 'N' no of gas molecules, each of mass m. Let G be the origin of 3 mutually perpendicular axes X, Y and Z. Let, u, v and w be the components of velocities along X, Y and Z-axis respectively. Furthermore, since the molecules are in random motion so we express in term of mean velocity $\bar{u}$, $\bar{v}$ and $\bar{w}$ along X, Y, Z-axis respectively. Thus mean velocity of gas molecules is,

$$\boxed{\bar{C}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2}$$

Since, the motion of gas molecules is in random, the mean square velocity of molecules along all direction is considered same. So

$\bar{u}^2 = \bar{v}^2 = \bar{w}^2$

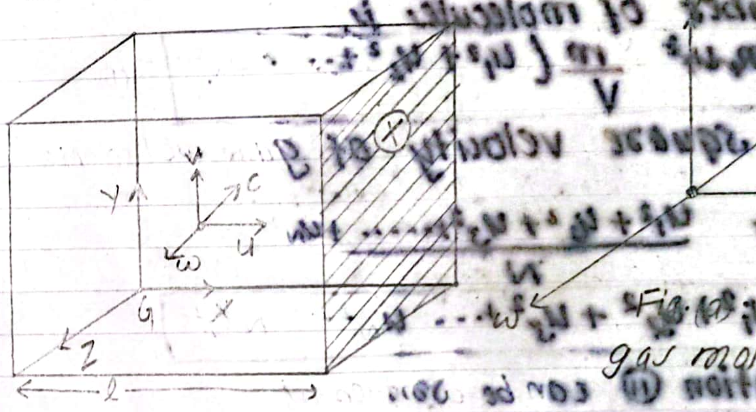


Fig: Motion of gas molecule in a cube

Consider force exerted on the face X is due to component u. Just before impact the momentum of the molecule due to u is mu and after striking it has momentum -mu, then change in momentum will be 2mu.

Time taken for two successive collision is given by, $\boxed{t = \frac{2l}{u}}$

and Force exerted by molecule on wall is

$\boxed{F = \frac{mu^2}{l}}$ [$\because F = \frac{\text{change in momentum}}{\text{time}}$

[illegible]

$$\boxed{P = \frac{mu^2}{V}}$$ [∵ V is volume of component cylinder]

[illegible] exerted by a molecule.

Let, $u_1, u_2 \dots u_N$ be the speeds of N molecules along a direction. Then, pressure exerted by N numbers of molecules is,

$$\boxed{P = \frac{m}{V}\left(u_1^2 + u_2^2 + \dots + u_N^2\right)} \quad -(i)$$

Mean square velocity of gas molecule,

$$\bar{u}^2 = \frac{u_1^2 + u_2^2 + u_3^2 + \dots + u_N^2}{N}$$

or $\boxed{u_1^2 + u_2^2 + u_3^2 + \dots u_N^2 = N\bar{u}^2}$

Equation (i) can be written as,

$$\boxed{P = \frac{mN\bar{u}^2}{V}}$$

or, $P = \frac{1}{3}\frac{mN}{V}\bar{c}^2$ [From (i)]

or, $P = \frac{1}{3}\frac{M}{V}\bar{c}^2$ [where, M is total mass of gas]

or, $\boxed{P = \frac{1}{3}\rho\bar{c}^2}$ [∵ $\rho = \frac{M}{V}$, is density of gas]

36. Average kinetic energy of a gas molecule is directly proportional to the absolute temperature of the gas.

[$K.E_{av} \propto T$]

From kinetic theory of gas,

→ The pressure exerted by gas of density d with root mean square $\bar{c}$ is given by,

$P = \frac{1}{3}d\bar{c}^2 = \frac{1}{3}\frac{M}{V}\bar{c}^2$ [∵ $d = \frac{M}{V}$ where M = Molar mass of ~~volume~~ gas, V = volume of gas]

or, $\boxed{PV = \frac{1}{3}M\bar{c}^2} \quad -(i)$

The equation of state for one mole of gas is given by,

$$PV = RT \quad -(ii)$$

From (i) and (ii),

or, $RT = \frac{1}{3}M\bar{c}^2$

or, $3RT = M\bar{c}^2$

or, $\boxed{\frac{3}{2}RT = \frac{1}{2}M\bar{c}^2}$

or, Here, $\frac{1}{2}M\bar{c}^2$ is the average kinetic energy of one mole. So, $\frac{3}{2}RT$ is the average kinetic energy per mole. If m is mass of a molecule and N is the number of molecule, then

$$\frac{3}{2}RT = \frac{1}{2}Nm\bar{c}^2$$

or, $\frac{3}{2}\frac{R}{N}T = \frac{1}{2}m\bar{c}^2$

or, $\boxed{\frac{1}{2}m\bar{c}^2 = \frac{3}{2}K_B T}$, where $K_B = \frac{R}{N}$ = Boltzmann's constant]

$\frac{1}{2}M\bar{c}^2$ is the average kinetic energy of a gas molecule. So,

$$K.E_{av} = \frac{3}{2}KT$$

$$\boxed{K.E_{av} \propto T}$$ Since $\frac{3}{2}K$ is constant.

[Proved]

37. To derive Boyle's law and Charle's law from pressure relation of kinetic theory of gases.

→ We have,

From kinetic theory of gase

$$\boxed{P = \frac{mn\bar{c}^2}{3V}}$$

where, P = Pressure on wall of container

m = mass of each molecule

n = no. of molecule

$\bar{c}^2$ = root mean square speed of gas molecule

V = volume of container.

Or, $P = \frac{1}{3}\frac{M\bar{c}^2}{V}$ [∵ M = mn] = Molarmass]

Or, $P = \frac{1}{3V} \times 2 \times \frac{1}{2}M\bar{c}^2$

Or, $PV = \frac{2}{3} \times \frac{3}{2}RT$ [∵ $\frac{1}{2}M\bar{c}^2 = \frac{3}{2}RT$]

Or, $\boxed{PV = RT}$ —①

Perfect gas:- Gas with no intermolecular forces of attraction between molecules & which strictly follows gases law.

We know,

Boyle's law states that, "at constant temperature pressure and volume of given mass of gas is inversely proportional with each other.

Then From eqⁿ①,

$PV = RT$

At constant temperature,

$PV = \text{constant}$

Or $P \propto \frac{1}{V}$, which is Boyle's law.

Again,

Charle's law of pressure states that at constant volume, pressure of given mass of gas is directly proportional to its temperature"

From ①,

~~At constant volume,~~

$PV = RT$

Or $P = \frac{R}{V}T$

At constant volume

Or $P = \text{constant} \times T$

Or $P \propto T$, which is charle's law of pressure

Similarly,

Charle's law of volume states that at constant pressure, volume of given mass of gas is directly proportional to its temperature."

From ①,

$PV = RT$

Or $V = \frac{R}{P}T$

At constant pressure,

$V = \text{constant} \times T$

Or $V \propto T$, which is Charle's law of volume.

38. Relation between Pressure coefficient (γ_v) and volume coefficient (γ_p).

Pressure coefficient:- It is defined as the change in pressure of certain mass of gas per unit pressure at 0°C per unit change in temperature at constant volume of gas.

Mathematically,

$$\gamma_v = \frac{P_\theta - P_0}{P_0 \cdot \theta}$$

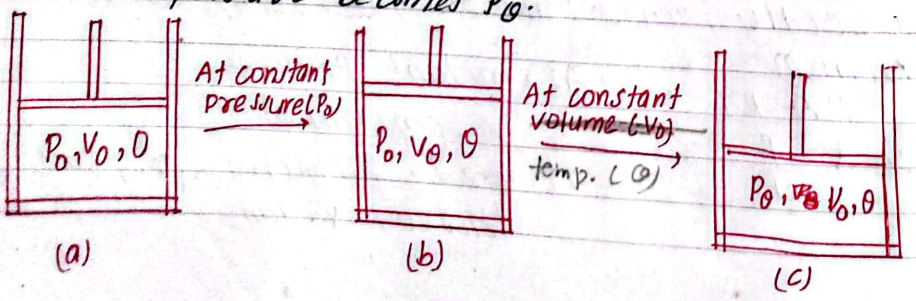
Volume coefficient: It is defined as the change in volume of certain mass of gas per unit volume at 0°C per unit degree change in temperature.

Mathematically,

$$\gamma_p = \frac{V_\theta - V_0}{V_0 \cdot \theta}$$

Proof:-

Let us consider a certain mass of gas having initial pressure 'P_0' and volume 'V_0' at 0°C. Then the gas is allowed to expand at constant pressure P_0 such that volume becomes V_θ and temperature θ°C. as shown in fig (b) and again the gas is compressed to its initial volume V_0 keeping temperature constant so that pressure becomes P_θ.



At first step,

From charles law of volume.

$$\boxed{V_\theta = V_0(1+\gamma_p\theta)} \quad \text{—(i)}$$

2nd step

From Boyles law,

$$\boxed{P_0 V_\theta = P_\theta V_0}$$

or $P_0 \cdot V_0(1+\gamma_p\theta) = P_\theta V_0$ [From (i)]

or $P_0(1+\gamma_p\theta) = P_\theta$

or $\boxed{P_\theta = P_0(1+\gamma_p\theta)}$ —(ii)

Also, from charles law of pressure.

$$\boxed{P_\theta = P_0(1+\gamma_v\theta)} \quad \text{—(iii)}$$

Comparing (ii) & (iii)

$$\boxed{\gamma_p = \gamma_v}$$

Hence, pressure coefficient is equal to volume coefficient if the gas follows gas laws.

6. First law of Thermodynamics

39. Relation between specific heat capacities (c_p & c_v) and Molar heat capacities (C_p & C_v)

To prove:-

$$C_p - C_v = R$$
$$c_p - c_v = r$$

→ When, the gas is heated at constant volume then supplied heat is used to rise internal energy (Temperature) of the gas. But, when the gas is heated at constant pressure, the heat supplied is used to do external work and rise internal energy (Temperature). So, more amout of heat is required to rise the temperature with same amount. Thus, gas have two values of molar specific heat capacities.

And, similar is the reason why $C_p > C_v$?

For, C_p, $dQ = dW + dU$ but,

for, C_v: $dQ = dU$

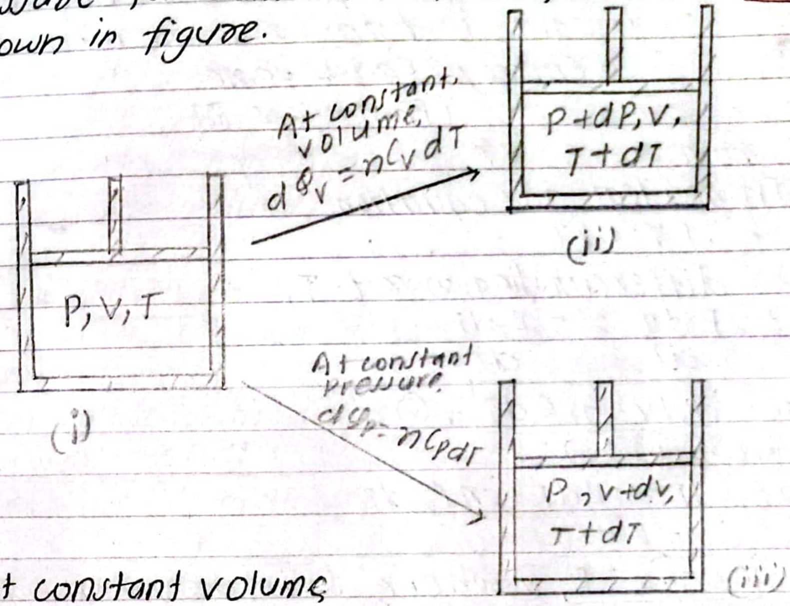
First law of thermodynamics states that, "When certain amount of heat is supplied to the system, then out of which some part is used to do external work and some part is used to increase the temperature"

Mathematically,

$$dQ = dU + dW$$
$$dQ = dU + PdV$$

Proof:-

Let us consider 'n' moles of ideal gas enclosed in a cylinder fitted with frictionless movable piston ~~pulley~~. Let P, V & T are initial pressure, volume and Temperature of the gas. The gas is heated separately at constant volume and constant pressure for same rise in temperature 'dT' as shown in figure.



* At constant volume

Amount of heat required to rise temperature by dT is given by, $\boxed{dQ_v = nC_v dT}$ —(i)

where, C_v is Molar heat capacity at constant volume

From first Law of Thermodynamics

$$dQ_v = dW + dU$$

Or, $dQ_v = dU$ [∵ At constant volume, $dW = 0$]

Or $\boxed{dU = nC_v dT}$ —(ii)

Similarly,

* At constant Pressure

Amount of heat required to rise temperature by dT is given by, $\boxed{dQ_p = nC_p dT}$ —(iii) where, C_p is molar heat capacity at constant pressure.

From first law of thermodynamics,

$dQ = PdV + dU$

or $\boxed{nC_p dT = PdV + nC_v dT}$ — (iv)

Since, temperature change dT is same in both case. So, internal energy remains same [From (iii) & (ii)]

We have,

From ideal gas equation,

$PV = nRT$

differentiating w.r.t. T,

$P\frac{dV}{dT} = nR\frac{dT}{dT}$

or $\boxed{PdV = nRdT}$ — (v)

From (iv) & (v),

$nC_p dT = nRdT + nC_v dT$

or $C_p = R + C_v$

or $\boxed{C_p - C_v = R}$ — (vi) where, R is universal gas constant.

Again,

dividing by molar mass 'M' on both sides of eq^n (vi),

$\frac{C_p}{M} - \frac{C_v}{M} = \frac{R}{M}$

$\boxed{c_p - c_v = r}$ $\quad [r = \frac{R}{M}]$

where, c_p and c_v are specific heat capacities at constant pressure and volume.

* PV^γ = constant
* Work done by adiabatic
* (Thermodynamic processes)
* " " isothermal

40. Equation of state for adiabatic process

→ Let us ~~condi~~ consider 'n' moles of gas enclosed in a cylinder having initial pressure, volume and temperature P, V and T.

From first law of thermodynamics,

$dQ = dU + dW$

[For adiabatic process, $dQ = 0$, $dU = nC_v dT$ and $dW = PdV$, where C_v = molar heat capacity at constant volume]

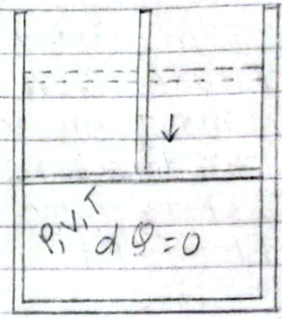


Fig: Adiabatic compression of gas.

So,

$\boxed{0 = nC_v dT + PdV}$ — (i)

For n moles of ideal gas, we have

$PV = nRT$

differentiating both sides,

or $PdV + VdP = nRdT$

or $\boxed{ndT = \frac{PdV + VdP}{R}}$ — (ii)

From (i) & (ii),

$\frac{(PdV + VdP)}{R} C_v + PdV = 0$

or $PdVC_v + VdPC_v + PdVR = 0$

or $PdV(C_v + R) + VdPC_v = 0$

or $PdV\,C_p + VdPC_v = 0$

dividing both sides by PVC_v,

or $\frac{PdVC_p}{PVC_v} + \frac{VdPC_v}{PVC_v} = \frac{0}{PVC_v}$

or $\gamma\frac{dV}{V} + \frac{dP}{P} = 0$

where, $\gamma = \frac{C_p}{C_v}$ is called ratio of heat capacities.

Integrating both sides

or, $\gamma\int\frac{dV}{V} + \int\frac{dP}{P} = $ constant

or, $\gamma \log_e V + \log_e P$ = constant

or, $\log_e V^\gamma + \log_e P$ = constant

or $\log_e (PV^\gamma)$ = constant

or $PV^\gamma = e^{\text{constant}}$

or $\boxed{PV^\gamma = \text{constant}}$

This expression gives the adiabatic relation between pressure and volume.

For Temperature & volume

$TV^{\gamma-1}$ = constant

For pressure and temperature

$P^{1-\gamma}T^\gamma$ = constant

41. Work done ~~by~~ in adiabatic process

$$W = \frac{nR}{\gamma - 1}(T_1 - T_2) = \frac{1}{\gamma - 1}(P_1V_1 - P_2V_2)$$

→ Let us consider 'n' moles of ideal gas enclosed in a cylinder with non-conducting walls fitted with weightless, frictionless and movable piston having initial pressure, volume and temperature P_1, V_1 and T_1 which finally changes to P_2, V_2 and T_2 on expansion.

Work done in adiabatic process is given by:-

$$W = \int_{V_1}^{V_2} P\,dV$$

For adiabatic process, $PV^\gamma = K$ (constant). Then,

$$W = \int_{V_1}^{V_2} \frac{K}{V^\gamma}\,dV$$

$$= K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= K\left[\frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1}\right]$$

$$= \frac{K}{1-\gamma}\left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right]$$

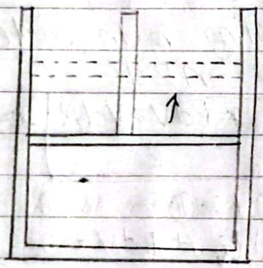


Fig: Adiabatic work done

Using, $P_1V_1^\gamma = P_2V_2^\gamma = \text{constant}(K)$,

$$\boxed{W = \frac{1}{1-\gamma}\left[\frac{P_2V_2^\gamma}{V_2^{\gamma-1}} - \frac{P_1V_1^\gamma}{V_1^{\gamma-1}}\right] = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1] = \frac{1}{\gamma-1}(P_1V_1 - P_2V_2)}$$ —(i)

From ideal gas equation, $P_1V_1 = nRT_1$ and $P_2V_2 = nRT_2$, so

$$\boxed{W = \frac{nR}{1-\gamma}(T_2 - T_1) = \frac{nR}{\gamma-1}[T_1 - T_2]}$$ —(ii)

which is the expression for work done by adiabatic ~~compression~~ process. (eqn (i) & (ii))

42. Work done in an isothermal process:

$$W = nRT\log_e\left(\frac{V_2}{V_1}\right) = nRT\log_e\left(\frac{P_1}{P_2}\right)$$

→ Let us consider 'n' moles of ideal gas enclosed in a cylinder having conducting walls fitted with piston. Then,

Total work done in expanding gas from initial state $A(P_1V_1)$ to $B(P_2V_2)$ is given by:

$$W = \int_{V_1}^{V_2} P\,dV \quad \text{—(i)}$$

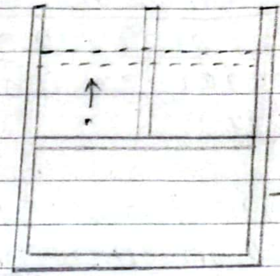


Fig: Isothermal work done.

For n moles of gas

$$PV = nRT \Rightarrow P = \frac{nRT}{V} \quad \text{—(ii)}$$

From (i) & (ii),

$$W = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV$$

$$= nRT[\log_e V]_{V_1}^{V_2} \quad [\because nRT \text{ is constant}]$$

$$= nRT[\log_e V_2 - \log_e V_1]$$

$$= nRT\log_e\left(\frac{V_2}{V_1}\right) \quad \text{—(iii)}$$

For isothermal process,

$$P_1V_1 = P_2V_2 \Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2} \quad \text{—(iv)}$$

From (iii) and (iv)

$$W = nRT\log_e\left(\frac{P_1}{P_2}\right) \quad \text{—(v)}$$

Equations (iii) and (v) are the expressions for work done in an isothermal process.

Thermodynamic Processes:

Any change which cause thermodynamic variables(P, V, T) to have a new set of values is called thermodynamic process.

Types:-

(i) Isochoric Process: Thermodynamic process that takes place at constant volume of the system. In this process gas is heated in non-expanding cylinder with fixed piston. Since, $dV=0$, so $dW=0$. Hence, $dQ=dU$ i.e. supplied heat is only used to increase internal energy.

(ii) Isobaric Process: It is the thermodynamic process in which pressure remains constant. So, gas is heated in expanding chamber slowly. And, $dQ = dU + nR(T_2 - T_1)$.

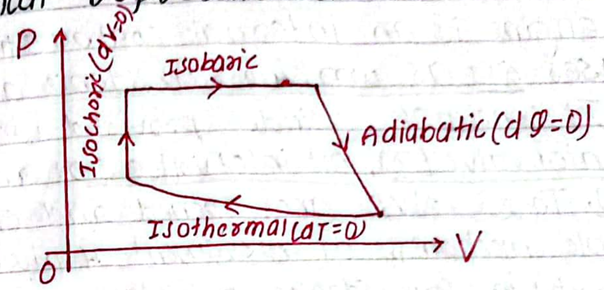
(iii) Isothermal process: It is the thermodynamic process in which temperature remains constant. In this process, exchange of heat between system and surrounding is done in such a way that the energy entering the system is equal to the energy transferred out of the system. Since, $dT=0$, $dU=0$. And $dQ=dW$. It is extremely slow process. Eg:- boiling water at constant temperature.

conducting

(iv) Adiabatic process: It is the thermodynamic process in which pressure, volume and temperature of the system changes without exchanging the heat between system and surrounding [$dQ=0$]. It is sudden and quick process. The wall of container must be insulated.

non-conducting

Graphical representation:-



7 Second law of Thermodynamics

~~43.~~ It ~~states th~~ has following statements:

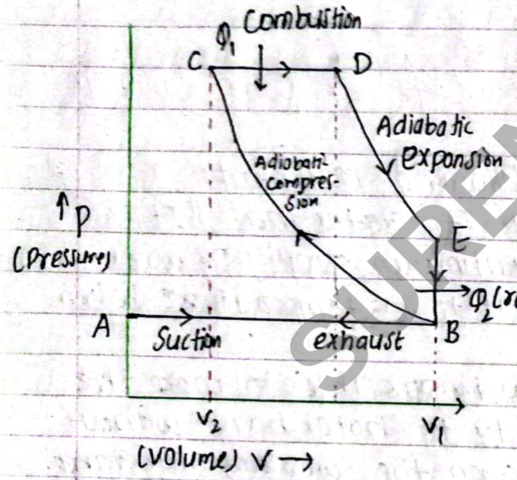
(i) Kelvin-Plank Statement: It states that, "It is impossible to get a continuous supply of work from a source by cooling it to a temperature below than its surrounding."

(ii) Clausius Statement: It is impossible to make the heat flow from colder body to hoter body without doing any external work on the working substance.

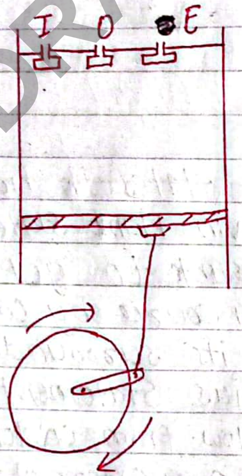
The second law of thermodynamics emphasizes the direction of heat flow. E.g. heat always flow from hot body to cold body, and never the reverse, unless external work is performed on the system.

43. Working of Diesel Engine

Diesel engine is an internal combustion engine which uses air as working substance and diesel as fuel. It consist of cylinder provided with three valves: inlet valve (I), Oil inlet valve (O) and Exhaust valve (E). These valves are opened and closed by suitable mechanism of movement of piston. The engine works in four stroke principle:



PV diagram



Diesel engine

(1) Suction stroke:-
In this stroke air inlet valve I is opened and other valves are closed. The piston moves down and pure air is sucked into the cylinder at atmospheric pressure. This is indicated by AB in PV diagram.

(2) Compression stroke:
All valves are closed. The piston moves up and compress the air adiabatically to about $\frac{1}{17}$ of initial volume, its temperature rises to 1000°C. This is indicated by BC in PV diagram.
At the end, i.e. at point C, the oil inlet valve 'O' is opened.

(3) Working stroke:-
The sprayed diesel in the cylinder burns due to high temperature inside cylinder which consumes Q_1 amount of heat. This is represented by CD in PV diagram.
At D when the temp. is about 2000K, the working substance expands adiabatically so temp. falls. This is represented by 'DE' in PV diagram.
At E, outlet valve is opened, so pressure immediately falls to atm. pressure by releasing Q_2 heat to the surrounding. which is represented by EB in PV diagram.

(4) Exhaust stroke:-
At last, residual burnt air is pushed out by inward motion of piston so volume reaches to zero at constant atm. i.e. represented by BA in PV diagram.

After that, the next cycle begins with fresh supply of air

Merits → (i) Cheap fuel
(ii) higher efficiency than petrol engine
(iii) Less inflammable, so risk of fire is reduced.

Demerits → (i) heavy so can't be used in light vehicles.

44. Working Of Petrol Engine:

Petrol engine is a four stroke internal combustion engine which consist of cylinder fitted with piston. Cylinder consist of inlet valve 'I', Outlet valve 'O' and spark plug 'SP' which are opened and closed by suitable mechanism of movable piston. It use 98% air and 2% petrol vapour as working substance.

(1) Suction stroke: In this stroke inlet valve 'I' is opened and outlet valve is closed. Air petrol mixture is sucked due to outward motion of piston which is represented by AB in P-V diagram.

(2) Compression stroke: Both valves are closed. The piston moves up and compresses the mixture adiabatically to about 1/5th of its original volume and temperature rises to 600°C. This is represented by BC in P.V diagram.

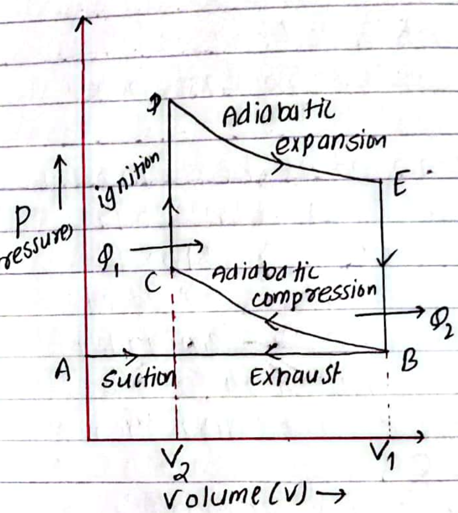
(3) Working Stroke: Both valves are closed. A spark is produced at sparking plug and the compressed mixture of air and petrol vapour ignites. The temperature rises to 2000°c and pressure of about 15 atm is developed in the cylinder. Working substance consumes 'Q_1' amount of heat due to ignition. This is represented by CD in PV diagram.

Now, the working substance expands adiabatically so that piston moves down. As a result, pressure and temp. drops. Only in this stroke, work is done by piston. This is represented by DE in PV diagram.

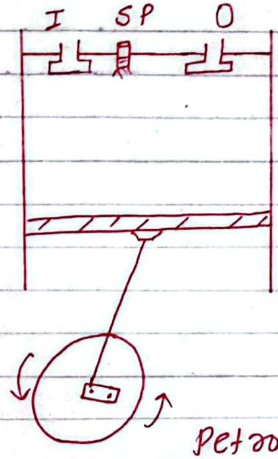
(4) Exhaust stroke:- The outlet valve 'O' is opened and the unburn heat 'Q_2' is rejected out to the surrounding such that pressure drops to atm. pressure. This is represented by EB in PV diagram.

At last, piston moves up and the burnt air is thrown out through the valve 'O'. This is represented by BA in PV diagram.

Then the next cycle occurs with a fresh air and petrol vapour.



P-V diagram of petrol engine



Petrol engine.

Merits → (i) Can be used in light vehicle
(ii) Practically smokeless
(iii) Effeciency is greater than steam engine.

Demerits → (i) Running cost is high
(ii) Higher risk of firing due to explosion of petrol vapour.
(iii) Less reliable due to occasional failure of spark plug.

Efficiency of heat engine:

It is defined as the ratio of the external work obtained to the heat energy absorbed by the working substance from the source. It is denoted by η.

$$\eta = \frac{\text{external work obtained}}{\text{heat energy absorbed from the source}}$$

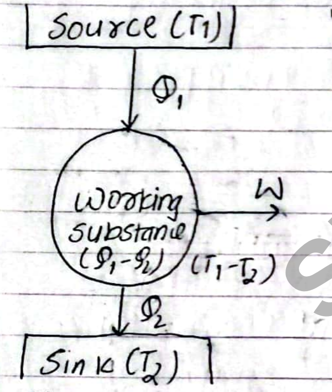


Fig: Block diagram of heat engine.

If Q_1 is amount of heat absorbed by working substance from source at high temperature (T_1), and Q_2 be amount of heat rejected to the sink at lower temperature (T_2). If W is mechanical work done by working substance then,

$$W = Q_1 - Q_2$$

$\therefore$ Efficiency of the engine,

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

No engine is 100% efficient, why?

→ $\eta = 100\%$ when $Q_2 = 0$ i.e. whole heat extracted from the source will be converted to mechanical work. No heat will be rejected to the sink. But, it is impossible to carry out any work without rejecting heat to the sink. So, efficiency of heat engine can never be 100%.

45. Experiment for determination of thermal conductivity of the solid. [Searle's experiment]

→ The experimental arrangement is shown in the figure below. A metal rod whose coefficient of thermal expansivity (K) is to be measured is placed inside a insulating frame covered with insulator. One end of rod 'x' is is connected with steam chamber where steam is passed continuously and other end is rounded with conducting pipe (copper pipe). Thermometers T_1 and T_2 are kept at a distance x in metal rod and mercury is kept in connection of thermometer to enhance conduction. Thermometers T_3 and T_4 are kept in hot reservoir and cold reservoir of copper pipe.

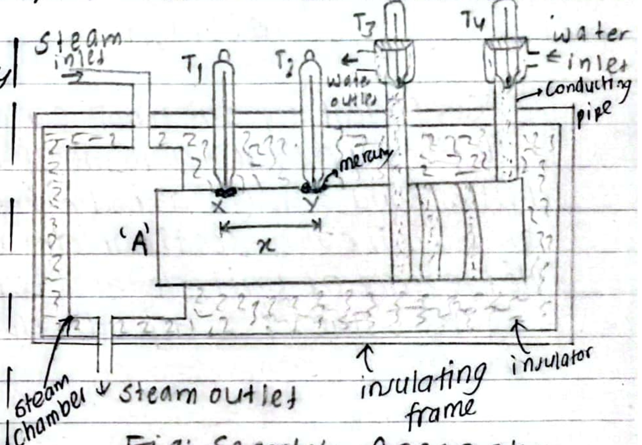


Fig: Searle's Apparatus.

When, one end is a heated by passing steam then, the heat is transferred to other end by conduction till steady state is reached. If 'A' is cross section area of rod and $\theta_1, \theta_2, \theta_3$ and θ_4 be temperature noted by thermometers T_1, T_2, T_3 and T_4 respectively. The frame prevent the loss of heat from metal surface.

Now, The rate of heat lost by rod is

$$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{x} \quad \text{or,} \quad \boxed{Q = \frac{KA(\theta_1-\theta_2)t}{x}} \quad \text{---(i)}$$

Heat gain by water at time t is

$Q = ms(\theta_3 - \theta_4)$ where m is mass of water collected out in time t and s is specific heat capacity of water.

Since, At steady state
Heat lost by metal = Heat gain by water

$$\frac{KA(\theta_1-\theta_2)t}{x} = mS(\theta_3-\theta_4)$$

$$\therefore \boxed{K = \frac{mS(\theta_3-\theta_4)x}{A(\theta_1-\theta_2)t}}$$

Hence, coefficient of thermal conductivity of the metal rod can be known by knowing parameters on the right hand side.

Coefficient of thermal conductivity (K)

$$K = \frac{Qx}{A(\theta_1-\theta_2)t}$$

Coefficient of thermal conductivity of a material is defined as the amount of heat flowing per unit second through a body having unit length and unit cross-sectional area with temperature difference of unit degree between two ends.

The value of K depends upon nature of material.

46. Stefan-Boltzmann's law of black body radiation.

It states that, " The amount of heat energy radiated per unit area per second of a perfectly black body is directly proportional to fourth power of absolute temperature."

If E be the amount of heat energy radiated per second per unit surface area of black body. Then

$$E \propto T^4$$

$$\frac{P}{A} = E \qquad \boxed{P = EA}$$

Or, $\boxed{E = \sigma T^4}$ —(i)

where, σ is the constant of proportionality called Stefan's constant and its value is $5.67 \times 10^{-8} \, Wm^{-2}K^{-4}$.

If P is radiant power by black body of area A then,

$\boxed{P = A\sigma T^4}$ —(ii)

For non-perfectly black body having emmissivity e, we have

$\boxed{E = e\sigma T^4}$ —(iii)

All above relations are valid when temperature of surrounding is absolute zero (OK). But, the surrounding has some temperature of T_0, the surrounding also radiates heat energy which is absorbed by black body. Hence, above equation was modified by Boltzmann. Then above equation is written as,

$\boxed{E = e\sigma(T^4 - T_0^4)}$ —(iv) [For non-perfectly black body.

And perfectly black body,

$\boxed{E = \sigma(T^4 - T_0^4)}$ —(v)

Above two equations are Stefan-Boltzmann's law.

Also,

$\boxed{P = e\sigma A(T^4 - T_0^4)}$

Unit: 4

Electrostatics

47. Electric field intensity due to charged sphere using Gauss's theorem at a point

(a) Outside the sphere

Let us consider a point P outside the sphere at a distance 'r' from centre where electric field intensity (E) has to be determined.

To use Gauss theorem we draw a spherical Gaussian surface passing through 'P' concentric with charged sphere. Then,

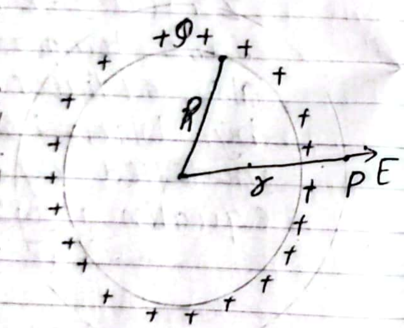
Area of Gaussian surface $(A) = 4\pi r^2$ — (i)

Then,

From definition, electric flux enclosed by the closed surface is

$\boxed{\phi = EA}$

or, $\phi = E 4\pi r^2$ — (ii)



And, From Gauss theorem,

$$\boxed{\phi = \frac{1}{\epsilon} \times q} \quad \text{— (iii)}$$

Equating (ii) and (iii),

$E \times 4\pi r^2 = \frac{1}{\epsilon} \times q$

or $$\boxed{E = \frac{1}{4\pi\epsilon} \times \frac{q}{r^2}}$$

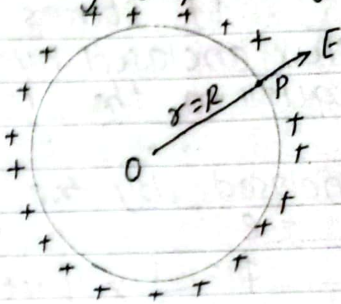
Thus, electric field intensity ~~due to~~ outside charged sphere is same as if whole charge were concentrated at its ~~sphere~~ centre.

(b) On the surface of the sphere

When point P lies at the point on the surface of the sphere, the Gaussian surface coincides with the sphere such that $r = R$. Thus, electric field can be found by replacing 'r' by 'R' in eq^n (ii).

i.e.

$$\boxed{E = \frac{1}{4\pi\epsilon} \frac{q}{R^2}} \quad \text{— (ii)}$$



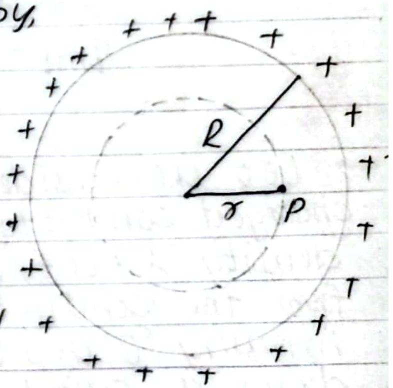
(c) On the point inside the sphere.

When point 'P' lies inside the sphere, then Gaussian surface lies inside the sphere. So, flux through the Gaussian surface is given by,

$\boxed{\phi = EA}$ — (iii)

Again from Gauss's theorem,

$$\phi = \frac{1}{\epsilon} q \quad \text{— (iv)}$$



But the net charge enclosed by Gaussian surface is zero as charge within the sphere is zero. i.e. $\boxed{q = 0}$

So, solving (iii) and (iv), we get $\boxed{E = 0}$

Thus, the total electric field inside the sphere is zero.

48. Electric field intensity due to line charge/linearly charged body using Gauss's theorem: $\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$

→ Gauss's Theorem:

It states that, "The total electric flux enclosed by the closed surface is equal to the $\frac{1}{\epsilon}$ times of total charge enclosed by the closed surface."

i.e.

$$\phi = \frac{1}{\epsilon} \times q_{net}$$

where, ϕ = Net/Total electric flux enclosed by closed surface

ϵ = Permittivity of the medium

q_{net} = Total electric charge enclosed by closed surface.

→ Let us consider an infinitely long thin charged conductor of uniform linear charge density 'λ'. Let, 'P' be a point at distance 'r' from the conductor where electric field intensity 'E' has to be determined. Let us draw a cylindrical Gaussian surface of length 'l' and radius 'r' enclosing 'P' as shown in figure.

Here, Electric lines of forces passes perpendicular to curved surface area but parallel to cross-section area. So, electric flux through the curved surface,

$$\phi = E \times A$$

or, $\boxed{\phi = E \times 2\pi r l}$ — (i) where, $A = 2\pi r l$ = curved surface area of cylinder.

And,

Net electric charge enclosed by Gaussian surface is

$\boxed{q = \lambda l}$ $[\because \lambda = \frac{q}{l}]$

From Gauss's theorem,

$$\phi = \frac{1}{\epsilon} \times q$$

or $\boxed{\phi = \frac{1}{\epsilon} \times \lambda l}$ — (ii)

where, ϵ is permittivity of medium

Equating (i) and (ii),

$$E \times 2\pi r l = \frac{1}{\epsilon} \times \lambda l$$

or, $$E = \frac{\lambda l}{\epsilon \times 2\pi r l}$$

or, $\boxed{E = \frac{\lambda}{2\pi \epsilon r}}$

(i.e. Electric field intensity only depend upon linear charge density but independent of total charge)

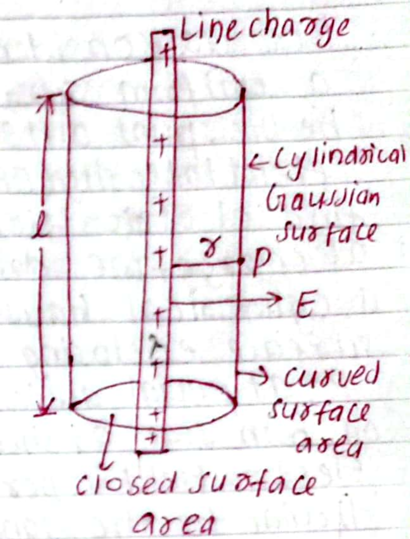


Fig: Electric field intensity due to line charge

49. Electric field Intensity due to infinite plane sheet of charge:

$$\boxed{E = \frac{\sigma}{2\epsilon}}$$

Let us consider an infinite plane sheet with a uniform surface charge density 'σ'. Let 'P' be the point outside the sheet where electric field intensity (EFI) 'E' is to be measured. To find electric field due to the plane sheet of charge, we draw a cylindrical Gaussian surface enclosing the P at equal distance on both side of sheet. Electric field is perpendicular to the plane sheet of charge and is directed in outward direction.

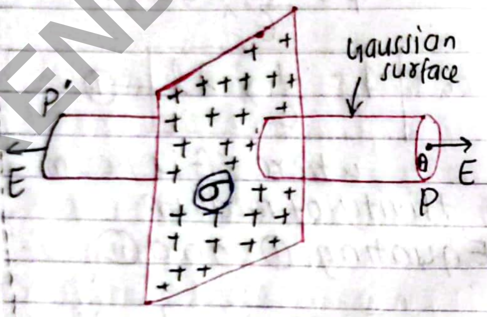


Fig: Infinite plane sheet of charge.

Total electric flux passing through the two end flat faces,

$$\boxed{\phi = 2 \times EA} \text{ — (i)}$$

Net charge enclosed by Gaussian surface,

$$q = \sigma \times A$$

By Gauss's theorem, we have

$$\phi = \frac{q}{\epsilon}$$

or $2 \times E \times A = \frac{\sigma \times A}{\epsilon}$ or $\boxed{E = \frac{\sigma}{2\epsilon}}$

i.e. EFI is independent of the distance from the sheet.

50. Electric field intensity due to charged conductor.

$$\boxed{E = \frac{\sigma}{\epsilon}}$$

Let us consider a charged plane conductor of surface charge density 'σ'. Let 'P' be the point above the conductor where electric field intensity 'E' has to be determined. To use Gauss's theorem we draw a cylindrical Gaussian surface enclosing the 'P' and having cross-section area 'A' drawn on a plane conductor normally as shown in figure. Then by symmetry electric field intensity at every point on the cross-section will also be 'E'.

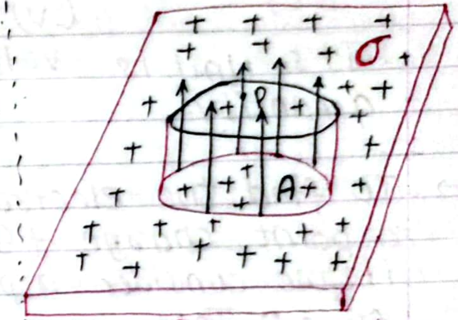


Fig: Field of the charged plane conductor.

Now,
From definition of electric flux (ϕ),

$$\boxed{\phi = E \times A} \text{ — (i)}$$

Net electric charge enclosed by the area 'A' is [Since, $\sigma = q/A$]

$$\boxed{q = \sigma A} \text{ — (ii)}$$

Applying Gauss theorem,

$$\phi = \frac{1}{\epsilon} \times q$$

or, $E \times A = \frac{1}{\epsilon} \times \sigma \times A$

or, $\boxed{E = \frac{\sigma}{\epsilon}}$ where ϵ is permittivity of the medium.

i.e. EFI outside the charged plain conductor is independent of total charge and distance parameter 'r'.

51. Electric potential at a point ~~near~~ due to point charge (near an electrostatic charge)

$$\boxed{V_p = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}}$$

→ Electric potential: Electric potential at a point in the electric field is defined as the amount of work done in bringing unit positive test charge from infinity to that point against the electrostatic force.

Electric potential (V) = $\frac{\text{Work done (W)}}{\text{Charge (q)}}$

It's unit is volt and it is a scalar quantity.

→ To find the electrostatic potential due to a point charge $+q$ at distance r from it, we consider a point ~~a~~ A at distance x from O. Then,

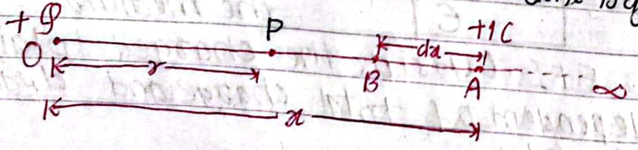
Electric field at that point is given by,

$$\boxed{E = \frac{q}{4\pi\epsilon_0 x^2}} \quad \text{—(1)}$$

where ϵ_0 is the permittivity of free space.

Then, small amount of work done to move unit positive test charge from A to B against electrostatic force through small distance 'dx' is given by,

$dW = -E \cdot dx$ [where, − sign indicates that work done is against F]



Now,
Total amount of work done in moving unit positive test charge from infinity to ~~distance~~ point P is obtained by,

$$W = \int_{\infty}^{P} dW = \int_{\infty}^{r} -E \cdot dx$$

or, $$W = -\int_{\infty}^{r} \frac{q}{4\pi\epsilon_0 x^2} \times dx$$

$$= -\frac{q}{4\pi\epsilon_0} \left\{ \int_{\infty}^{r} x^{-2} dx \right.$$

$$= -\frac{q}{4\pi\epsilon_0} \times -\left(\frac{1}{x}\right)_{\infty}^{r}$$

$$\boxed{W = \frac{q}{4\pi\epsilon_0 r}}$$

From definition, work done in bringing +1C charge from infinity to point P is the electric potential at point P.

i.e.

$$V_p = \frac{W}{1} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}$$

$$\therefore \boxed{V_p = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r}}$$

52. Expression for the potential difference between two points in electric field due to point charge

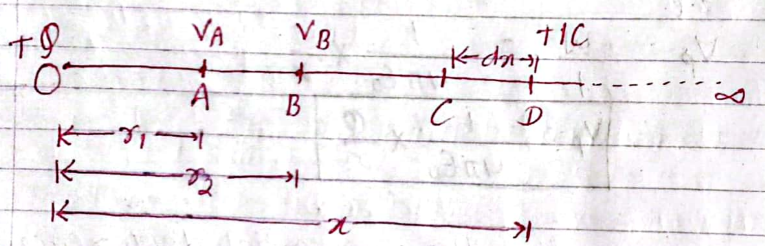
→ Electric potential: It is the amount of work done in an electric field to bring a unit positive test charge from infinity to the given point.

Electric potential (V) = $\frac{W}{q_0}$ and SI unit is J/C or volt (V).

Potential gradiant: The rate of change of electric potential w.r.t. distance is called potential gradiant. It is vector quantity and denoted by $\frac{dV}{dx}$.

→ Electric potential difference between two points in an electric field is the amount of work done in bringing unit positive test charge from one point to another point against electrostatic force.

→ Let '+Q' be the point charge at point O in a free space whose electric field is extended upto infinity. Let A and B be two points in the electric field between which electric potential difference has to be determined at distance 'r_1' and 'r_2' from O.



Let +1C charge be at point D at distance x from O. Then electrostatic force at point D is given by

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}} \quad \text{(i)}$$

If dW is the small amount of work done to move +1C charge ~~from~~ by dx against the field then,

$$dW = -F dx$$

$$\boxed{dW = -\frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}dx} \quad \text{(ii)}$$

[-ve sign indicates that work done is against electric field]

Now,
Amount of work done in bringing +1C charge from B to A is given by integrating eqn (ii), i.e.

$$W_{BA} = \int_B^A -dW = \int_{r_2}^{r_1} -\frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}dx$$

$$= +\frac{Q}{4\pi\epsilon_0}\int_{r_2}^{r_1} x^{-2}dx$$

$$= \frac{+Q}{4\pi\epsilon_0}\left[\frac{1}{x}\right]_{r_2}^{r_1}$$

$$= \frac{+Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\boxed{W_{BA} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \quad \text{(iii)}$$

From definition,
Work done in bringing unit positive test charge from one point to another point is potential difference between two points. So,

$$\boxed{V_{AB} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

which is the required expression.

53. Relation between potential gradiant and electric field intensity at a point

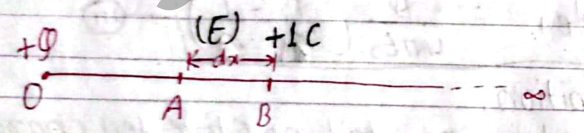
$$\boxed{E = -\frac{dV}{dr}}$$

→ Let us consider two points A and B separated by small distance 'dx' in the electric field of charge +Q. Since, A and B are close to each other electric field intensity (E) between them is constant.

Then,
work done to move +1C charge from B to A is given by

$\boxed{W = -E\,dx}$ — (i) [∵ For 1C F = E]

Negative sign shows work done is against electrostatic force.



If dV is the potential difference between A and B then,
Work done to move +1C charge from B to A is given by

$\boxed{W = dV}$ — (ii)

From (i) and (ii)

$-E\,dx = dV$

or $\boxed{E = -\frac{dV}{dx}}$

Negative sign shows that electric field is directed towards decreasing potential.

Gravitation

54. Expression for the escape velocity on the surface of the earth

$(V_e = \sqrt{2gR} = \sqrt{2}\,V_0)$

→ Let us consider earth as a perfect sphere of mass 'M' and radius 'R' and a body of mass 'm' is lying on the earth. For that body to escape from gravitational pull of the earth, the body should be projected with velocity 've' such that kinetic energy of the body at surface of the earth must be equal to the amount of work done in moving that body from surface of the earth to infinity.

i.e.

$$\frac{1}{2} m v_e^2 = \frac{GMm}{R}$$

or, $\boxed{v_e = \sqrt{\frac{2GM}{R}}}$ — (i)

or $v_e = \sqrt{\frac{2gR^2}{R}}$ ∵ $[GM = gR^2]$ as $g = \frac{GM}{R^2}$

∴ $\boxed{v_e = \sqrt{2gR}}$ — (ii) $\boxed{V_e = \sqrt{2} \times \sqrt{gR} = \sqrt{2}\,V_0}$

[∵ $V_0 = \sqrt{gR}$] when $R+h \to R$

~~Where~~ Eqⁿ (i) and (ii) are required expression of escape velocity.

Escape velocity:-
The minimum velocity with which a body must be projected upward in order to overcome the gravitational pull so that it escape into space is called escape velocity.

55. Expression for the time period of the satellite revolving around the earth and height of satellite from earth surface.

→ Let us consider a satellite of mass 'm' revolving around the circular orbit of radius 'r' at height 'h' from earth surface such that the earth is considered a perfect sphere of mass 'M' and radius 'R'.
We know,
Time period is the total time taken by a satellite to revolve around the orbit.
Thus,

$$\text{Time period } (T) = \frac{\text{Circumference of orbit}}{\text{orbital velocity } (v_o)}$$

$$\text{or, } T = \frac{2\pi r}{v_o}$$

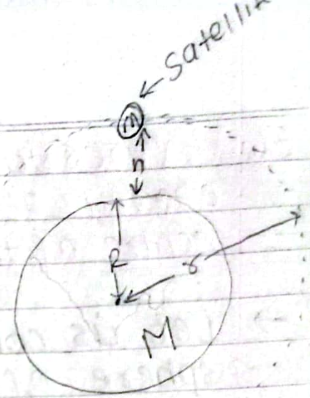
$$\text{or } T = \frac{2\pi r}{\sqrt{\frac{GM}{R+h}}} \quad \left[\because v_o = \sqrt{\frac{GM}{R+h}}\right]$$

$$\text{or } \boxed{T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}} \text{ — (i)}$$

$$\text{or } \boxed{T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}} \text{ (ii)} \quad [\because GM = gR^2]$$

Equations (i) and (ii) are expressions for time period of the satellite.
Again,
Height of satellite above earth surface:-
We have,

$$T = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$



Squaring both sides,

$$\text{or } T^2 = \left(\frac{2\pi}{R}\right)^2 \times \frac{(R+h)^3}{g}$$

$$\text{or } T^2 = \frac{4\pi^2 \times (R+h)^3}{gR^2}$$

$$\text{or } (R+h)^3 = \frac{T^2 gR^2}{4\pi^2}$$

$$\text{or } R+h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3}$$

$$\text{or } \boxed{h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3} - R} \text{ — (iii)}$$

Equation (iii) is the expression for height of satellite above earth surface.

Geostationary satellite:-
It is the satellite which seems to be stationary when viewed from a point on earth surface.
Conditions required for a satellite to be geostationary:-
(i) Time period of revolution of satellite must be equal to rotation of earth i.e. 24hr.
(ii) Direction of revolution of satellite should be same direction of rotation of earth i.e from west to east.
For Geostationary satellite:-
$T = 24\text{hrs} = 86400\text{sec}$, $g = 9.8\text{m/s}^2$, $R = 6.4\times10^6\text{m}$

$$\text{So, } h = \left[\frac{(86400)^2 \times 9.8 \times (6.4\times10^6)^2}{4\pi^2}\right]^{1/3} - 6.4\times10^6$$

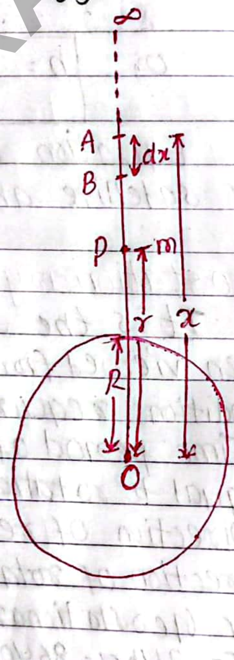
$$= 36000000\text{m}$$

$$= 36000\text{km}$$

Note: orbit of geostationary satellite are called parking orbits.

56. Expression for the gravitational potential energy of a body at a distance r from the centre of the earth.

→ Let us consider the earth as a perfect sphere of mass M and radius R. Let 'P' be the point at the distance 'r' from center 'O' where Gravitational potential energy of a body of mass m is to be determined. For that, let us consider a point A at a distance 'x' from center of the earth. Then, the gravitational force experienced by the body at that point is given by:-



$$\boxed{F = \frac{GMm}{x^2}} \quad \text{(i)}$$

Then small amount of work done to displace the body from A to B by small displacement dx is given by,

$$dW = F dx$$

$$\boxed{dW = \frac{GMm}{x^2} dx} \quad \text{(ii)}$$

Then, total amount of work done in bringing a body from infinity to point P is given by:-

$$W = \int_{\infty}^{P} dW = \int_{\infty}^{r} \frac{GMm}{x^2} dx$$

$$= Gmm \left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -GMm \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\boxed{W = -\frac{GMm}{r}}$$

This work done is equal to gravitational potential energy at point P.

Then,

Gravitational potential energy is given by

$$\boxed{U = -\frac{GMm}{r}}$$

[As Gravitational potential energy is the amount of work done in bringing a body from infinity to that point]

Negative sign indicates that gravitational force is attractive in nature and maximum value of P.E. is 0 when $r \to \infty$.

Total energy of satellite

The total energy of satellite revolving around the earth is the sum of potential energy and kinetic energy.

i.e.

$$E = P.E. + K.E.$$

K.E. of satellite is due to orbital velocity of satellite

$$\text{i.e. } K.E. = \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} m \frac{GM}{r} \quad \left[\because v_0 = \sqrt{\frac{GM}{r}}\right]$$

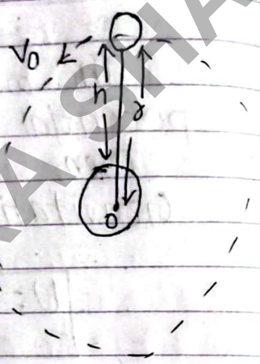
$$\boxed{K.E. = \frac{GMm}{2r}} \quad \text{(i)}$$

[r = radius of orbit of satellite & m = mass of satellite]

P.E. on the satellite is due to gravitational force between earth and satellite and it is given as.

$$P.E. = -\frac{GMm}{r}$$

Then total energy is given by



$$E = K.E. + P.E.$$
$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$
$$= \frac{GMm - 2GMm}{2r}$$
$$\boxed{E = -\frac{GMm}{2r}}$$

-ve sign indicates that gravitational force is attractive in nature and the satellite is bound to earth.

** Derivation of acceleration due to gravity see from book or copy.

Elasticity

58. Energy stored in Stretched wire

$$\boxed{E = \frac{1}{2} Fx}$$

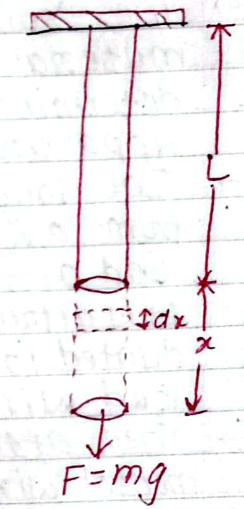
Let us consider a wire of length 'L' ~~suspended~~ suspended from a rigid support. Let a force 'F' is applied at its lower end to produce extension 'x' as shown in figure.

If dw is the small amount of work done to produce small extension dx, keeping F constant. Then

$$\boxed{dw = Fdx} \text{ —(i)}$$

If wire obeys Hooke's law, Then

$\boxed{F = kx}$ (ii), where K = Force constant



Then, Total amount of work done to produce extension x is given by,

$$W = \int dw = \int F dx$$
$$= \int_0^x kx\,dx$$

$$\boxed{W = \frac{1}{2} kx^2} \text{ —(iii)}$$

Since, $F = kx$, so

$$W = \frac{1}{2}(kx)\cdot x$$

This work done is stored as the potential energy of the stretched force.

$$\boxed{W = \frac{1}{2} Fx \quad \text{i.e.} \quad W = \frac{1}{2} \times \text{stretching force} \times \text{extension}} \text{ —(iv)}$$

Eqn (iv) is required expression.

Energy density: It is defined as elastic potential energy per unit volume & denoted by 'U'.

Then, Energy density (U) $= \frac{\frac{1}{2}Fx}{V} = \frac{\frac{1}{2}Fx}{AL} = \frac{1}{2}\left(\frac{F}{A}\right)\times\left(\frac{x}{L}\right)$

$$\therefore \boxed{U = \frac{1}{2} \times \text{stress} \times \text{strain}}$$

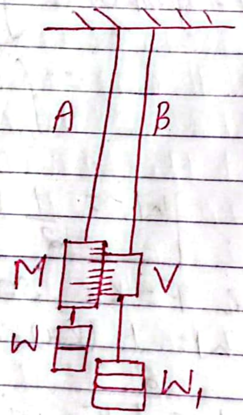
59. ~~The~~ Determination of Young's modulus of elasticity

→ Young's modulus of elasticity (Y) is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.

$$Y = \frac{F/A}{\Delta l/L} = \frac{FL}{A\Delta L}$$

→ The Young's modulus of elasticity Y of a material ~~can~~ in the form of wire can be determined in laboratory by using vernier apparatus as shown in fig.

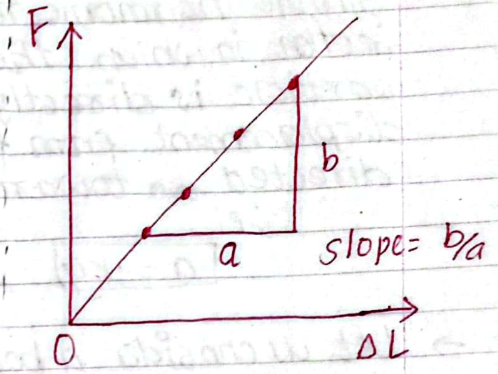
→ Two long thin wires, A and B are suspended from a rigid support. At the end of the wire A, a scale M is attached which is graduated in mm and small load W is attached to it.



→ Then, at the end of experimental wire B, a vernier scale V is attached which can slide over reference scale M.

→ The initial length 'L' of the wire B is noted and its diameter 'd' is measured with the help of micrometer screw gauge. Then test wire B is loaded (typically upto 100N in five steps) and the resulting extension is noted as a function of the load.

→ Then the graph is plotted between the load and the extension of wire B as shown in fig. The slope of the straight line in the graph is measured now.



~~Tensile stress of~~

Young's modulus (Y)

$$= \frac{\text{Tensile stress}}{\text{Tensile Strain}}$$

$$= \frac{F/A}{\Delta L/L}$$

$$= \frac{FL}{\Delta L \times A}$$

i.e. $\boxed{Y = \text{slope} \times \frac{L}{A}}$

where, F = applied load

A = Area of cross-section of wire

ΔL = extension of wire

L = original length.

Note: Definitions, Moduli's, elastic after effects, Stress, Strain, fatigue, Poisson's Ratio, hysteresis, Hooke's law, read from book.

Periodic Motion

60. Expression for total energy of simple harmonic oscillator

→ Simple harmonic motion is a oscillatory motion in which the acceleration of the particle is directly proportional to the displacement from the mean position and directed ~~to~~ towards the mean position.

i.e.

$\boxed{a = -Ky}$ K is constant.

→ Let us consider a body of mass 'm' executing simple harmonic motion with angular velocity 'ω', amplitude 'r' and 'x' be the displacement. Then we have

The small amount of work done by force F to produce small displacement dx is given by $\boxed{dW = -Fdx}$, where -ve sign showns that F and dx are oppositely directed.

Now, Total amount of work done is obtained as

$$W = \int_0 dw = \int_0^x -F dx$$

$$= -\int_0^x (ma)\, dx$$

$$= -\int_0^x m(-\omega^2 x)\, dx \quad [\because a = -\omega^2 x \text{ in S.H.M.}]$$

$$= m\omega^2 \left[\frac{x^2}{2}\right]_0^x$$

$$\therefore \boxed{W = \frac{1}{2} m\omega^2 x^2} \quad \text{—(i)}$$

This work done is stored as P.E.

$$\therefore \boxed{P.E. = \frac{1}{2} m\omega^2 x^2} \quad \text{—(ii)}$$

For Kinetic energy (K.E.),

$$K.E. = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m(\omega\sqrt{r^2 - x^2})^2 \quad [\because v = \omega\sqrt{r^2 - x^2}]$$

$$\therefore \boxed{K.E. = \frac{1}{2} m\omega^2 (r^2 - x^2)} \quad \text{—(iii)}$$

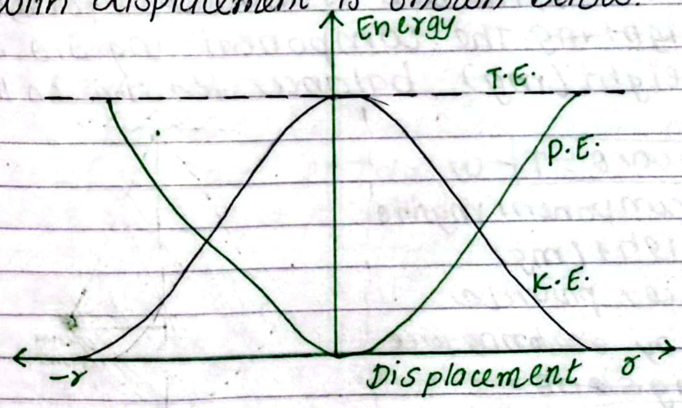
Now,

$$\text{Total energy} = T.E. = K.E. + P.E.$$

$$= \frac{1}{2} m\omega^2 (r^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

$$\boxed{T.E. = \frac{1}{2} m\omega^2 r^2} \quad \text{—(iv)}$$

Equation (iv) shows that the total energy of a particle executing S.H.M. remains constant throughout the motion. The variation of P.E., K.E. and T.E. with displacement is shown below:



Time period:

$$\omega = \frac{2\pi}{T}$$

$$\therefore T.E. = \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 \times r^2$$

$$\text{or, } T = \sqrt{\frac{4\pi^2 m r^2}{2T.E.}}$$

$$\therefore \boxed{T = \pi r \sqrt{\frac{2m}{T.E.}}}$$

61. Simple pendulum.

Simple pendulum is the heavy point mass suspended by an inextensible, weightless and flexible string from the rigid support.

Let us consider a small bob of mass 'm' suspended by string of effective length 'l'. Let, A be the equilibrium position of bob. When bob is displaced by small angle θ to new position B and then released, then it starts to oscillate.

At position B, the bob is under two forces, (W & T)

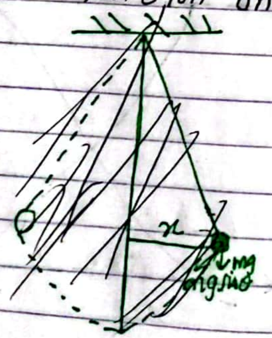
(i) ~~Weight mg~~ The component $mg\cos\theta$ of its weight (mg) balances tension on the string.

i.e. $\boxed{mg\cos\theta = T}$ —(i)

(ii) The component $mg\sin\theta$ of weight (mg) ~~balances~~ provide necessary restoring force.

i.e. ~~$mg\sin\theta =$~~

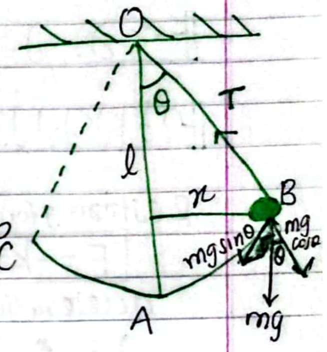
i.e. $\boxed{F = -mg\sin\theta}$ —(ii)



-ve sign indicates that restoring force is opposite to displacement.

If 'a' be the linear acceleration of the metallic bob, Then,

$\boxed{F = ma}$ —(iii)



From (ii) and (iii),

$-mg\sin\theta = ma$

or, $\boxed{a = -g\sin\theta}$

For small angle, $\sin\theta \approx \theta$, then

$\boxed{a = -g\theta}$

From figure,

$$\theta = \frac{\overset{\frown}{AB}}{l}$$

or $\boxed{\theta = \frac{x}{l}}$ [For small angle, $AB \approx x$ is linear displacement]

Then,

$$a = -g \times \frac{x}{l}$$

or $\boxed{a = -\frac{g}{l} \times x}$ —(iv)

or $a \propto x$ [$\because g/l$ is constant]

Since, acceleration is directly proportional to linear displacement from mean position and − sign shows that it is directed opposite to displacement. Hence motion of simple pendulum is simple harmonic.

Further,

For S.H.M, $\boxed{a = -\omega^2 x}$ —(v)

From (iv) and (v)

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

We know, Time period of simple pendulum (T) is given by,

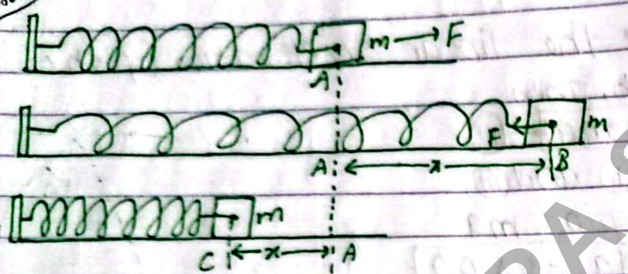
$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}}$$ —(vi)

Eqn (vi) is required expression for time period of simple pendulum.

62. Mass spring system

Quick Derivation

Horizontal



Restoring force

$$\boxed{F = -Kx} \quad \text{—(i)}$$

acceleration produced by restoring force

$$a = \frac{F}{m} = \frac{-Kx}{m}$$

$K \to$ Force constant

or $\boxed{a = -\frac{K}{m}x}$ —(ii) $\quad x \to$ extension.

$\boxed{a \propto x} \to$ SHM

For S.H.M, $\boxed{a = -\omega^2 x}$ —(iii)

Equating (ii) & (iii)

$$\omega = \sqrt{\frac{K}{m}}$$

Time period of oscillation (T)

$$\omega = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}}$$

Vertical

After mass m is attached

$\boxed{F_1 = -Kl}$ —(i) $\quad \boxed{F_1 = mg}$

When force is applied to pull spring

$\boxed{F_2 = -K(l+y)}$ —(ii)

Net restoring force producing oscillation

$F = F_2 - F_1$

$F = -K(l+y) + Kl$

$\boxed{F = -Ky}$ —(iii)

If a is acceleration produced,

$\boxed{F = ma}$ —(iv)

From (iii) & (iv)

$ma = -Ky$

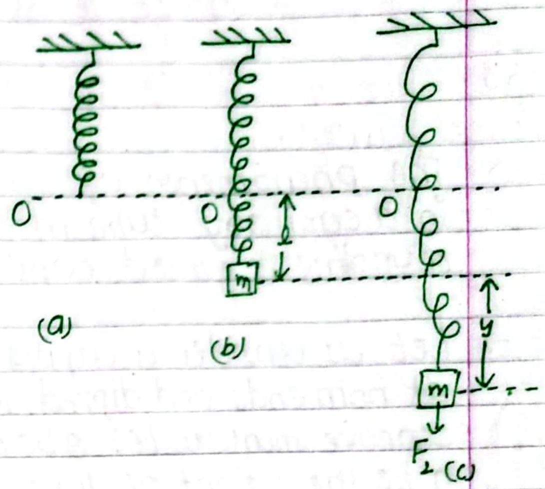
or $\boxed{a = -\frac{K}{m}y}$ —(v)

$\boxed{a \propto y}$ — S.H.M.

For S.H.M, $\boxed{a = -\omega^2 y}$ —(vi)

From (v) & (vi),

$\boxed{\omega = \sqrt{\frac{K}{m}}}$ & $\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$



Fluid Mechanics

Surface Tension

63. Rise of Liquid in capillary tube (Ascent formula)

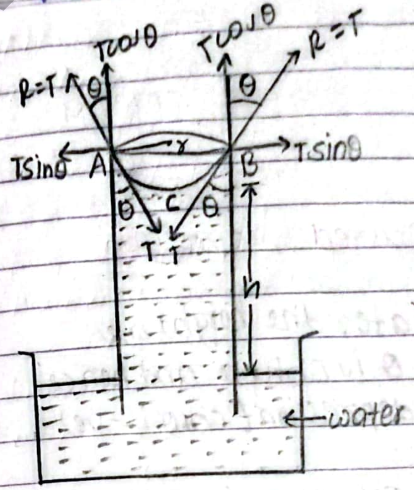
→ The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surrounding is called capillary action or capillarity.

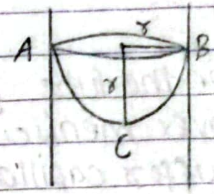
→ Let us consider a capillary tube of radius 'r' open at both ends and dipped into a liquid which has concave meniscus. Let θ be the angle of contact and h be the height of liquid rises, ρ be the density of liquid and T be surface tension of liquid. The surface tension forces causes the liquid to exert a downward directed force 'T'. The force 'T' acts along the tangent at the point of contact A. From ~~net~~ Newton's 3rd law, the tube exerts equal and opposite reaction 'R'. The reaction ($R=T$) can be ressolved into two components $T\sin\theta$ and $T\cos\theta$ along horizontal and veotical respectively. The horizontal component cancel each other whereas vertical components are added which pulls the liquid upward. Component '$T\cos\theta$' act along whole circumference of meniscus.

$$\therefore \boxed{\text{Total upward force} = T\cos\theta \times 2\pi r}$$

Volume of the liquid above the free surface is given by:

V = Volume of cylinder of height h and radius r + Volume of cylinder of height r and radius r − Volume of hemisphere of radius r





$$\text{Or, } V = \pi r^2 h + \pi r^2 r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^2 h + \pi r^3 - \frac{2}{3}\pi r^3$$

$$= \pi r^2 h + \frac{1}{3}\pi r^3$$

$$\text{or, } \boxed{V = \pi r^2\left(h + \frac{r}{3}\right)}$$

For equilibrium

$\boxed{\text{Total upward force} = \text{Weight of the liquid column}}$

$$T\cos\theta \times 2\pi r = \text{mass of liquid column} \times g$$

$$\text{or, } T\cos\theta \times 2\pi r = \rho \times V \times g$$

$$\text{or, } T\cos\theta \times 2\pi r = \rho \times \pi r^2\left(h + \frac{r}{3}\right) \times g$$

$$\text{Or, } \boxed{T = \frac{\rho r\left(h + \frac{r}{3}\right)g}{2\cos\theta}}$$

$$\text{Or, } T = \frac{\rho r h g}{2\cos\theta}$$

[For very fine tube, $\frac{r}{3}$ may be neglected)

$$\therefore \boxed{T = \frac{rh\rho g}{2\cos\theta}}$$

or $$\boxed{h = \frac{2T\cos\theta}{r\rho g}} \text{ — (1)}$$

Equation (1) is required expression

Note:-

(i) Narrower the tube greater the height rise.

(ii) For convex meniscus θ is obtuse and hence liquid suffer capillary depression ($\cos\theta$ is -ve).

64. Relation between surface tension and surface energy of a liquid

Let us consider a rectangular frame of wire ABCD. When frame is dipped in soap solution, the wire BC is pulled toward left due to surface tension. If 'T' be the surface tension of film, and 'l' be the length of wire BC then force of surface tension on BC is given by,

$$\boxed{F = T\times 2l} \text{ — (1)}$$

Since film is in contact with 2 surface of wire so total length of contact is $2l$. If wire is moved outward to distance 'x' from BC to B'C'. The work has to be done against force of surface tension.

Then, Work done in increasing surface area (W)

$= \text{Force} \times \text{distance}$

$= (T\times 2l)\times x$

$= T(2lx)$

$\therefore$ $\boxed{W = \text{Surface Tension} \times \text{increase in surface area.}}$

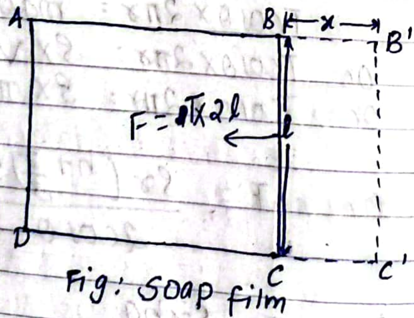
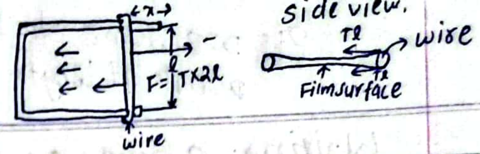


Fig: Soap film

For understanding,



Then,

Surface energy (σ) = $\frac{\text{Work done to increase surface area}}{\text{Increase in surface area}}$

$$= \frac{T\times 2lx}{2lx}$$

or, $\boxed{\sigma = T}$

i.e. Surface energy is numerically equal to the surface tension.

Chapter → 12.3

Viscocity

65. Poiseuille's Formula

(Derivation by Dimensional Analysis)

Let us consider a liquid flowing steadily through a horizontal pipe of length 'l' and radius 'r'. Poiseuille found that the volume of a liquid flowing through a capillary tube per second depends upon:

(i) the pressure gradient $\left(\frac{P}{l}\right)$ (i.e. rate of change of pressure with length)

(ii) radius of capillary tube, r

(iii) coefficient of viscosity of liquid, η

That is, Volume of liquid flowing per second

$$\boxed{V \propto \left(\frac{P}{l}\right)^a r^b \eta^c}$$ where

a, b, c are constants to be determined.

$$\therefore \boxed{V = k\left(\frac{P}{l}\right)^a r^b \eta^c} \text{ — (i)}$$

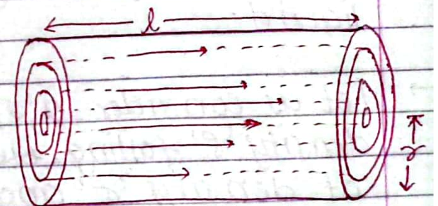


Fig: Streamlined flow of viscous liquid.

Proportionality constant k is dimensionless.

P is pressure difference of two point at length l $\boxed{P = P_2 - P_1}$ not pressure at that point.

Writing above equation in terms of dimension

$$[L^3T^{-1}] = [ML^{-2}T^{-2}]^a \cancel{[ML^{-1}T^{-1}]} [L]^b [ML^{-1}T^{-1}]^c$$

or $[M^0L^3T^{-1}] = [M^{a+c}L^{-2a+b-c}T^{-2a-c}]$

Equating power of M, L & T, we get

$a + c = 0$ —(i)

$-2a + b - c = 3$ —(ii)

$-2a - c = -1$ —(iv)

Solving above equations we get

$a = 1,\ b = 4$ and $c = 1$.

Substituting these values in eqⁿ (i),

$$V = \frac{k\,P r^4}{l\eta}$$

Experimentally it is found that, $k = \pi/8$, so

$$\boxed{V = \frac{\pi P r^4}{8 l \eta}}$$ which is required formula

66. Stoke's ~~law~~ method to determine coefficient of viscosity of a liquid and terminal velocity:

→ Terminal velocity (v):- It is a constant velocity acquired by a body while falling in the viscous liquid (fluid)

→ Let us consider a spherical ball of radius 'r', density 'ρ' falling ~~under~~ inside the viscous liquid of density 'σ' and coefficient of viscosity 'η' as shown in figure.

At first velocity of the ball increases which increase the upward viscous force and finally a stage comes in when the weight of the ball acting vertically downward is ~~equal~~ balanced by sum of upthrust and upward viscous force. In this condition net force acting on ball is zero and ball falls with constant velocity 'v' known as terminal velocity.

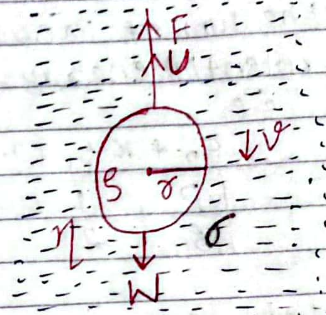


Fig: Spherical ball falling in viscous liquid.

Here,

Weight of spherical ball (W) = mass of ball × g

$= \rho \times V \times g$

$$\boxed{W = \frac{4}{3}\pi r^3 \rho g} \text{ —(i)}$$

Upthrust (U) = $V\sigma g$

$$\boxed{U = \frac{4}{3}\pi r^3 \sigma g} \text{ —(ii)}$$

& From Stokes law,

Upward viscous force $\boxed{(F) = 6\pi\eta r v}$ —(iii)

When the ball acquired terminal velocity 'v' then,

$W = U + F$

or, $6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$

or, $6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$

or $\eta = \frac{4}{18} \frac{r^2(\rho-\sigma)g}{v}$

or, $$\boxed{\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v}} \text{ —(iv)}$$

And,

$$\boxed{v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}} \text{ —(v)}$$

Equation (iv) and (v) are required expressions.

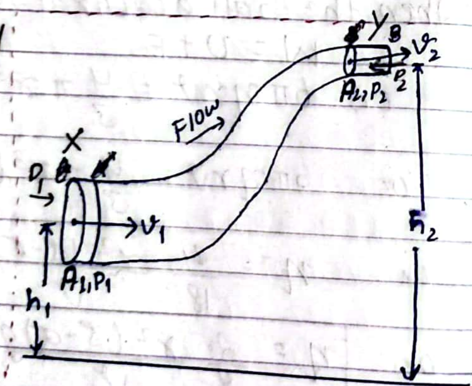
67. Bernoulli's theorem for the steady flow of an incompressible and non-viscous fluid.

→ It states that, " For a streamlined flow of a ideal liquid (non-viscous and incompressible) the sum of pressure energy, kinetic energy and potential energy per unit mass is always constant."
i.e.

$$E_p + K.E. + P.E. = \text{constant}$$

$$\boxed{\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}}$$

→ Let us consider the streamlined flow of ideal liquid through the tube XY as shown in the figure. Let, P_1, A_1, V_1, h_1 and P_2, A_2, V_2, h_2 be the pressure, cross section area of tube, velocity of flow of liquid and height at the end X and Y respectively. Then,



Force applied on liquid due to pressure P_1 at X is, $\boxed{F_1 = P_1 A_1}$

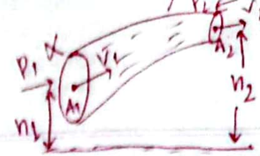
displacement of liquid at time t due to pressure P_1 at X is

$$\boxed{S_1 = v_1 t}$$

Then,
Work done on liquid due to the pressure P_1 at X $(W_1) = F_1 S_1$

$$\boxed{\therefore W_1 = P_1 A_1 v_1 t}$$

Similarly,
work done by liquid against the pressure P_2 at Y $\boxed{(W_2) = P_2 A_2 V_2 t}$



∴ Net work done by liquid due to pressure to flow from X to Y is

$$W = W_1 - W_2$$

$$\boxed{W = P_1 A_1 v_1 t - P_2 A_2 v_2 t} \quad -①$$

Then, From equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\boxed{A_1 v_1 t = A_2 v_2 t = V}$$ where, V is volume of the liquid flow in time t.

As, $A_1 > A_2$ so, $v_1 < v_2$

Then, From eq^n ①,

$$W = P_1 V - P_2 V$$

or, $W = (P_1 - P_2) V$

or, $\boxed{W = (P_1 - P_2)\frac{m}{\rho}}$ $\left[\because V = \frac{m}{\rho}\right.$ & ρ is density of liquid

The work done is used to give kinetic energy and potential energy to the fluid. so,

$$\boxed{\text{Increase in K.E.} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$$ [As, $v_2 > v_1$]

$$\boxed{\text{Increase in P.E} = mgh_2 - mgh_1}$$ [As, $h_2 > h_1$]

From work-energy theorem,
Work done by pressure energy = Increase in K.E + Increase in P.E.

or $(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$

or $\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gh_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gh_2$

or $\boxed{\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}}$

which proves Bernoulli's theorem

Optics

Lens

68. **Len's maker's formula:-**

$$\left(\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right)$$

→ Let us consider a convex lens of focal length 'f' and radii of curvature $C_1O = R_1$ and $C_2O = R_2$. Refracting angle of prism (A) is formed drawing two tangents from point M and N.

From figure,

$\angle MPN = A$ = angle of prism

$KO = h$ = height

$\angle NKC_2 = \delta$ = angle of deviation

$\angle KC_1O = \alpha$

$\angle KC_2O = \beta$

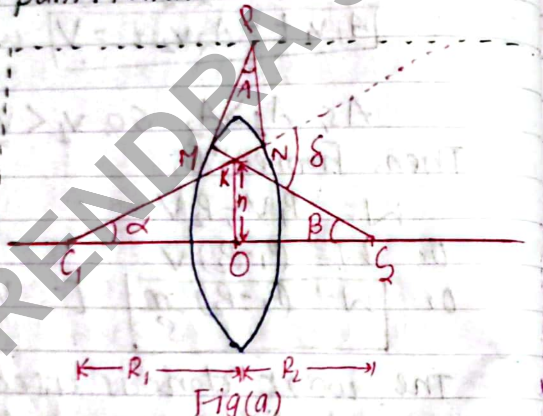


Fig (a)

By geometry,

$$\boxed{A = \alpha + \beta} \quad \text{—(i)}$$

Also,

$\tan\alpha = \frac{h}{R_1}$ & $\tan\beta = \frac{h}{R_2}$

But α & β are very small. So, we can write

$$\boxed{\alpha = \frac{h}{R_1}} \text{ and } \boxed{\beta = \frac{h}{R_2}}$$

Putting these values in eqⁿ (i).

$$A = \frac{h}{R_1} + \frac{h}{R_2}$$

$$\therefore \boxed{\frac{A}{h} = \frac{1}{R_1} + \frac{1}{R_2}} \quad \text{—(ii)}$$

→ When ray PQ is incident parallel to the lens at the same height h. After reflection it passes through principle focus.

Now. The deviation produced by lens is given by

$$\boxed{\delta = (\mu-1)A} \quad \text{—(iii)}$$

In right angled ΔQFO,

$\tan\delta = \frac{h}{f}$

For small angle, $\tan\delta \approx \delta$

$$\therefore \boxed{\delta = \frac{h}{f}} \quad \text{—(iv)}$$

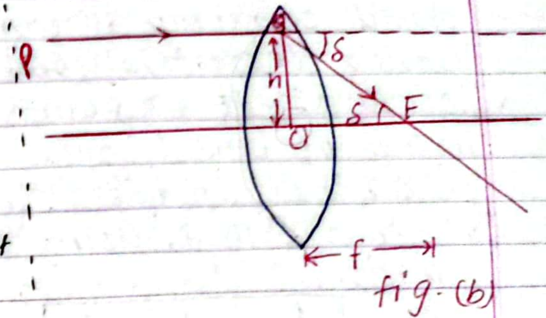


fig. (b)

From (iii) and (iv), we get

$$\frac{h}{f} = (\mu-1)A$$

Or. $\frac{h}{A}$ ⇒ $$\boxed{\frac{A}{h} = \frac{1}{f(\mu-1)}} \quad \text{—(v)}$$

Now, From (ii) & (v).

$$\frac{1}{f(\mu-1)} = \frac{1}{R_1} + \frac{1}{R_2}$$

Or, $$\boxed{\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

This equation is known as lens maker's formula

Sign convention for radius of curvature:-

(i) Radius of curvature of any lens is measured from optical centre.

(ii) Radius of curvature for convex lens is taken positive and radius of curvature of concave lens is taken negative.

(iii) Radius of curvature of plane surface is taken infinity (∞).

69. Formula for the focal length to two thin lenses in contact $\left(\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\right)$

→ Let us consider two thin lenses L_1 and L_2 placed coaxially in contact with each other having respective focal lengths 'f_1' and 'f_2'.

→ Let at first a point object O is placed in front of lens L_1 only at distance 'u' from C so that image 'I'' is formed at distance 'v'' from C. Then, using lens formula for lens 'L_1',

$$\boxed{\frac{1}{f_1}=\frac{1}{u}+\frac{1}{v'}} \text{ — (i)}$$

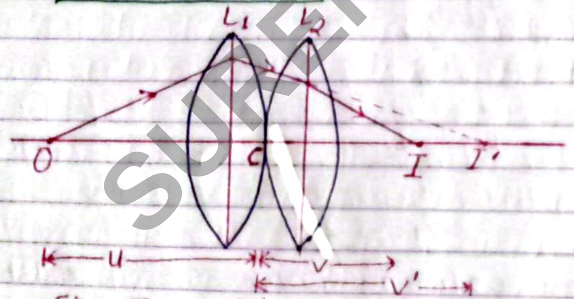


Fig: Two thin lenes in contact.

→ When lens L_2 is kept in contact with lens L_1, then I' formed by L_1 acts as virtual object for lens L_2 whose final image is formed at 'I' at distance 'v' from C.

For lens L_2,

$$\boxed{\frac{1}{f_2}=\frac{1}{-v'}+\frac{1}{v}} \text{ — (ii)}$$

Adding (i) & (ii).

$$\boxed{\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{u}+\frac{1}{v}} \text{ — (iii)}$$

If 'F' is the combined focal length of two lenses then, from lens formula

$$\boxed{\frac{1}{F}=\frac{1}{u}+\frac{1}{v}} \text{ — (iv)}$$

From (iii) & (iv),

$$\boxed{\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}}$$

F → equivalent focal length.

Note:- For 'n' lenses kept in combination

$$\boxed{\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\cdots+\frac{1}{f_n}}$$

Similarly, power of combination is.

$$\boxed{P=P_1+P_2+P_3+\cdots+P_n}$$

The ability of convex lens to converge and concave lens to diverge the ray of light falling on it is called power of lens (P). It is reciprocal of focal length of lens expressed in metre.

The point on a principle axis where the rays of light coming parallel to principle axis either converge or appear to diverge from it is called principle focus (f).

Optical instruments

70 Expression for magnifying power of compound microscope.

→ The optical instrument which is used to observe highly magnified image of extremely tiny object is called compound microscope. It consists of two convex lenses arranged coaxially on a tube. The lens which is near the object with small aperture and small focal length is called objective lens 'O' and lens near the eye having large aperture and moderate focal length is called eye piece 'E'.

When final image is formed at near point (D):-

→ Let AB be the small object lying between F_o and $2F_o$ of objective lens which forms magnified, real and inverted image A'B' beyond $2F_o$. The eyepiece is placed behind the image and it is so adjusted that a virtual and highly magnified image A"B" ~~is~~ of object A'B' forms at least distance of distinct vision from the eye. The focus of eye piece lies close from the A'B'. Magnification of eye piece is same as magnification produced by simple microscope.

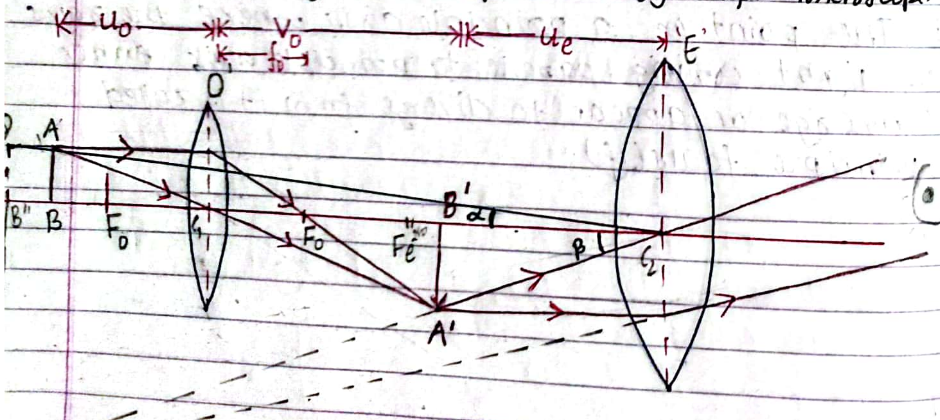


Fig: compound Microscope

Magnification produced by simple microscope
$(m) = -\left(1+\frac{D}{f_e}\right)$

When final image is formed at least distance of distinct vision:-

Magnifying power: The ratio of visual angle by the image to the visual angle by the object on eye is called magnifying power.

i.e. $\boxed{M = \frac{\beta}{\alpha}}$ —(I)

From given fig,

$\beta = \angle A''C_2B''$ = angle substended by image A"B" at D distance.

$\alpha = \angle QC_2B''$ = angle substended by object AB at D distance.

Then. As α and β are very small [object seen in microscope is very small and image is formed far from eye piece)

$\boxed{\beta \approx \tan\beta = \frac{A''B''}{B''C_2}}$

$\boxed{\alpha \approx \tan\alpha = \frac{QB''}{B''C_2}}$

Putting values of α & β in eq^n(I),

$$M = \frac{A''B''}{B''C_2} \times \frac{B''C_2}{QB''}$$

or, $M = \frac{A''B''}{A'B'} \cdot \frac{A'B'}{AB}$ [Multiplying and Dividing by A'B']

or, $\boxed{M = m_e \cdot m_o}$ —(II)

$m_e = \frac{A''B''}{A'B'}$ = Magnification produced by eye piece.

$M_o = \frac{A'B'}{AB}$ = Magnification produced by objective lens.

As we know magnification produced by eye piece is same as magnifying power of simple microscope, so

$$\boxed{m_e = -\left(1+\frac{D}{f_e}\right)}$$ —(III)

For magnification of objective lens (m_o)

We know, $\boxed{m_o = \frac{v_o}{u_o}}$

For objective lens from lens formula,

$$\frac{1}{f_o} = \frac{1}{u_o} + \frac{1}{v_o}$$

[Multipling by v_o on both sides)

or, $\boxed{\frac{v_o}{u_o} = \frac{v_o}{f_o} - 1}$

or, $\boxed{m_o = \frac{v_o}{f_o} - 1}$ — (IV)

From (II), (III) & (IV),

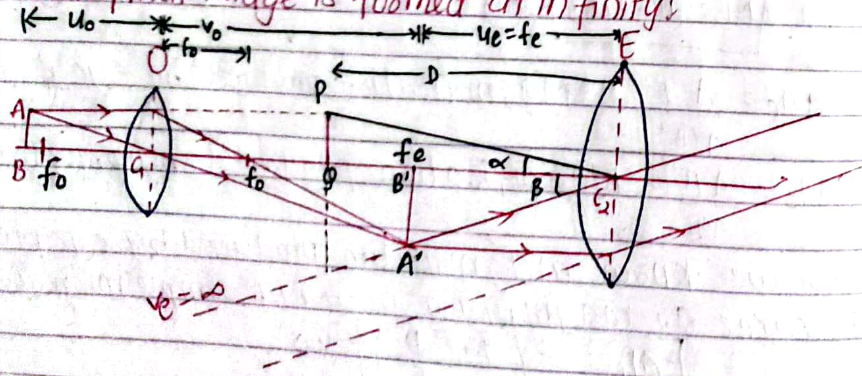
$$\boxed{M = -\left(1 + \frac{D}{f_e}\right)\left(\frac{v_o}{f_o} - 1\right)}$$

which is required expression for magnifying power of compound microscope and -ve sign indicates that image formed is virtual.

Note:- For this condition,
Length of microscope or distance between two lens is given by

$$\boxed{d = L = v_o + u_e}$$

When final image is formed at infinity:



$$M = \frac{\beta}{\alpha} = \frac{A'B'}{B'C_2} \times \frac{QC_2}{ABQ}$$ [As $\alpha = \frac{PQ}{QC_2} = \frac{AB}{QC_2}$ & $\beta = \frac{A'B'}{B'C_2}$]

or, $M = \frac{A'B'}{AB} \times \frac{QC_2}{B'C_2}$

or, $M = M_o \times \frac{-D}{f_e}$ [$QC_2 = -D$ (virtual image)]

or, $M = \frac{v_o}{u_o} \times \frac{-D}{f_e}$

$\boxed{\text{For objective lens, } \frac{v_o}{u_o} = \frac{v_o}{f_o} - 1}$

So,

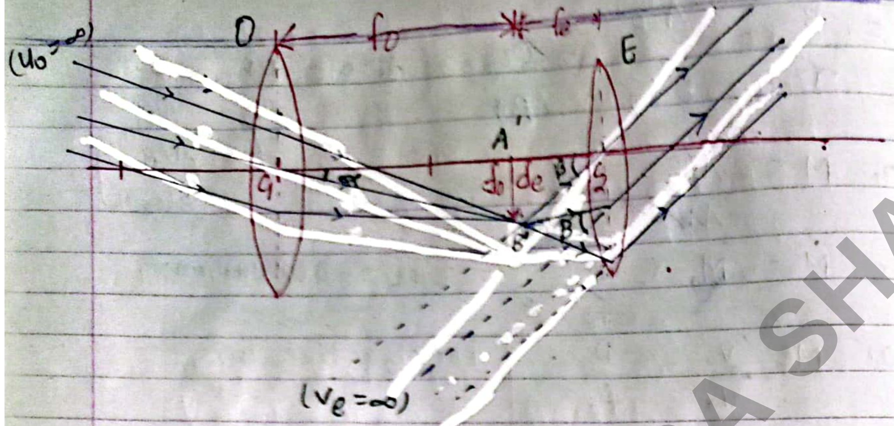
$$\boxed{M = \frac{-D}{f_e}\left(\frac{v_o}{f_o} - 1\right)}$$ which is required expression

Note:- For this condition,
$\boxed{d = L = v_o + f_e}$

7.2. Structure and Working of an astronomical telescope.

→ An astronomical telescope is an optical instrument which is used to see the ~~magnified and~~ clear image of distant heavenly bodies.
The final image formed by astronomical telescope is always virtual, ~~erect~~ inverted and magnified.
The objective lens has large focal length and large aperture whereas eyepiece has small focal length and small aperture. Both lenses are mounted coaxially in a tube.
On the basis of position of final image it can be adjusted in two positions.
(1) Normal adjustment (when the final image is formed at infinity)

[Read defect of vision from book or copy, causes, Remedy.]



When the parallel beam of light coming from the infinite object is incident on the objective lens 'O', the real image A'B' is formed on its focal plane. The image A'B' is supposed to be object for eye piece. The eye piece is so adjusted that A'B' lie at focal length 'f_e' of eye piece and the final magnified image is formed at the infinity as shown in figure.

Magnifying power in normal adjustment
It is defined as the ratio of angle substended by image 'β' to the angle substended by object 'α' seen directly when both lies at infinity.

i.e.

$$\boxed{M=\frac{\beta}{\alpha}} \quad \text{(1)}$$

For very small α and β from figure

$$\boxed{\tan\alpha \approx \alpha = \frac{A'B'}{C_1A'}}$$

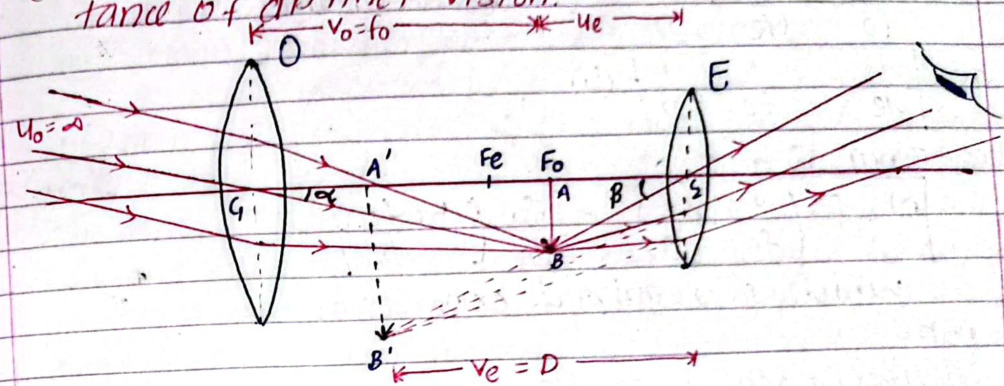
$$\boxed{\tan\beta \approx \beta = \frac{A'B'}{C_2A'}}$$

Putting value of α and β in (1),

$$\boxed{M=\frac{C_1A'}{C_2A'}=\frac{f_o}{f_e}}$$ which is required expression.

Note: Distance between two lenses (L) = f_o+f_e

(2) When the final image is formed at least distance of distinct vision:



When the parallel beam of light coming from infinite object is incident on the objective lens a real image AB is formed on focal plane. The image AB is supposed to be object for eye piece. The eye piece is so adjusted that ~~the~~ AB lies between F_e and C_2 and final highly magnified image A'B' is formed at least distance of distinct vision from eye piece.

Magnifying power:
It is defined as the ratio of angle substended by final image ~~to the~~ at least distance of distint vision 'β' to the angle substended by object at eye at infinity, when seen directy 'α'.

i.e. $$\boxed{M=\frac{\beta}{\alpha}} \quad \text{(i)}$$

For small α and β, from figure,

$$\boxed{\tan\alpha \approx \alpha = \frac{AB}{C_1A}} \qquad \boxed{\tan\beta \approx \beta = \frac{AB}{C_2A}}$$

Thus, $$\boxed{M=\frac{C_1A}{C_2A}=\frac{f_o}{u_e}} \quad \text{(ii)} \quad \text{[From fig]}$$

If fe be focal length of eyepiece,

$$\frac{1}{f_e} = \frac{1}{v_e} + \frac{1}{u_e}$$

or, $$\frac{1}{f_e} = \frac{1}{u_e} - \frac{1}{D} \quad [\because v_e = -D \text{ (virtual)}]$$

or, $$\boxed{\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}} \quad \text{(iii)}$$

From (i) and (iii)

$$\boxed{M = f_o\left(\frac{1}{f_e} + \frac{1}{D}\right) = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

which is required expression.

Note:
Tube length $(L) = f_o + u_e$.

Electrostatics
4.
Capacitance and Dielectrics.

73. Expression for energy stored in the capacitor. $\boxed{U = \frac{1}{2}CV^2}$

→ Capacitance:
Capacitance of capacitor is the capacity to hold the charge. It is also defined as the ratio of amount of charge given to the capacitor to the potential difference developed across its plates.

i.e. $\boxed{C = \frac{Q}{V}}$

Unit of capacitance:

$C = \frac{q}{V} = \frac{\text{coulomb (C)}}{\text{volt (V)}} = C/V = \text{Farad (F)}$

Farad is the large unit of capacitance. So, we use the small units

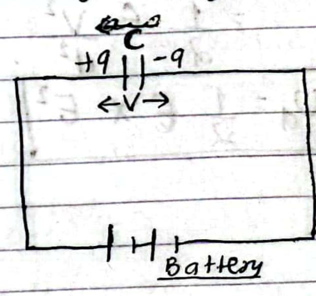
(i) Pico farad (PF) → $1\,PF = 10^{-12} F$
(ii) Micro farad (μF) → $1\,\mu F = 10^{-6} F$
(iii) Nano farad (nf) → $1n\,F \rightarrow 10^{-9} F$

→ (When a capacitor is charged through a battery, it charges to some extent and the further charging is opposed by the electric field setup between the plates. To further charge the capacitor, battery has to do the work against developed electric field. This work done is stored in the capacitor in the form of electric potential energy.)

→ Let us consider a capacitor of capacitance C has zero charge initially. After connection to battery it acquires '+q' charge at potential difference V. Then small amount of work done by a battery to charge the capacitor by 'dq' charge is given by:

$$\boxed{dW = V\,dq} \Rightarrow \boxed{dW = \frac{q}{C}\,dq} \quad [\because V = \frac{q}{C}]$$

Thus, total work done in delivering charge 'q' to the capacitor is given by:



$$W = \int_0^q \frac{q}{C}\,dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$\therefore \boxed{W = \frac{1}{2C} \times q^2}$$

This work done is stored in the capacitor as the electric potential energy U.

$$\therefore \boxed{U = \frac{1}{2}\frac{q^2}{C}}$$

Since, $q = CV$, so we can write

$$\boxed{U = \frac{1}{2} qV}$$

Also, $\boxed{U = \frac{1}{2} CV^2}$

which is required expression.

Energy density

The energy stored per unit volume of the space between the plates of capacitor is called its energy density.

Let us consider the parallel plate capacitor each of area 'A' separated by distance 'd' with the dielectric medium of permittivity 'ϵ', then the energy density of the capacitor is given by:

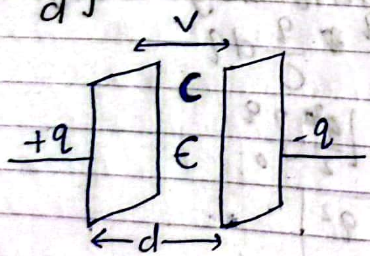
$$\text{Energy density } (U_d) = \frac{\text{Energy stored}}{\text{Volume}}$$

$$= \frac{1}{2} CV^2 \times \frac{1}{A \times d}$$

$$= \cancel{\frac{1}{2} CV^2} = \frac{1}{2} \times \frac{A\epsilon}{d} \times \frac{V^2}{Ad} \quad \left[\because C = \frac{A\epsilon}{d}\right]$$

$$= \frac{1}{2} \epsilon \frac{V^2}{d^2} \quad \left[\because E = \frac{V}{d}\right]$$

$$\boxed{U_d = \frac{1}{2} \epsilon \times E^2}$$



74. Expression for equivalent capacitance of capacitors.

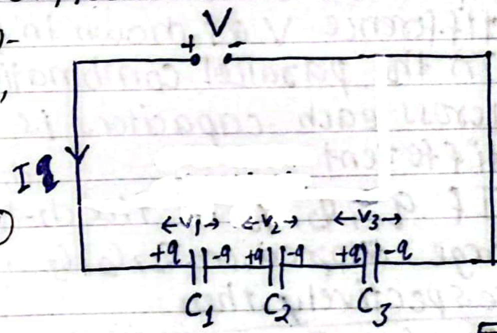
→ The grouping of two or more capacitors in an electric circuit is known as combination of capacitors.

(1) Series combination of capacitors

→ Let us consider the three capacitors having capacitance C_1, C_2 and C_3 respectively are connected in series with the battery having potential difference 'V' as shown in figure.

In series combination charge on each capacitor is same but potential difference is divided.

If V_1, V_2, V_3 be the potential difference across C_1, C_2, C_3 respectively then,



$$\boxed{V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}} \quad \text{— (i)}$$

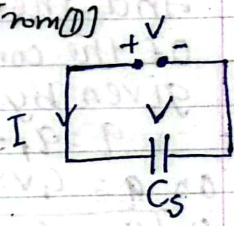
$$\boxed{I = \frac{q}{t}}$$

And total potential difference of the combination is:- $V = V_1 + V_2 + V_3$

$$\text{or } V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \quad [\text{from (i)}]$$

$$\text{or } V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\text{or } \boxed{\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad \text{— (ii)}$$



If C_S is equivalent capacitance of series combination

Then, $\boxed{C_S = \frac{q}{V}}$ — (iii)

From (ii) and (iii),

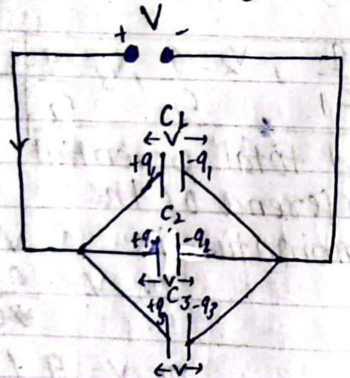
$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For n-capacitor in series

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

i.e. In the series combination of the capacitor, reciprocal of the equivalent capacitance is equal to the sum of reciprocal of the capacitance of individual capacitors

(ii) Parallel combination of capacitor:
Let us consider the three capacitors having capacitance C_1, C_2, C_3 respectively connected in parallel with the battery having potential difference V as shown in figure.
In the parallel combination potential difference across each capacitors is same but charge is different.
If q_1, q_2, q_3 be the charge stored on C_1, C_2, C_3 respectively, then



$$\boxed{q_1 = C_1V, q_2 = C_2V, q_3 = C_3V} \text{—(i)}$$

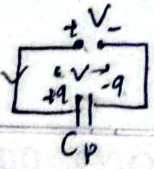
And the total charge of the combination is given by:

$$q = q_1 + q_2 + q_3$$

or, $q = C_1V + C_2V + C_3V$ [from (i)]

or, $$\boxed{\frac{q}{V} = C_1 + C_2 + C_3} \text{—(ii)}$$

If C_p be the equivalent capacitance of the parallel combination, then $\boxed{C_p = \frac{q}{V}}$ —(iii)

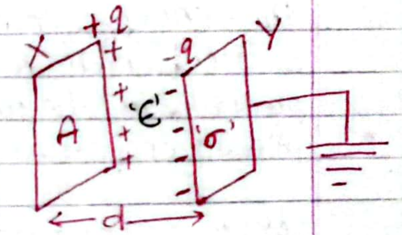


From (ii) and (iii),

$$\boxed{C_p = C_1 + C_2 + C_3}$$

i.e. In parallel combination of capacitor equivalent capacitance is equal to the sum of capacitance of individual capacitance.

75. Capacitance of parallel plate capacitor:
Parallel plate capacitor is one of the simplest form of capacitor. It consists of two parallel plates X and Y having each of area 'A' separated by the distance 'd' with the dielectric medium of permittivity 'ε'.
Let, '+q' charge is given to the plate X due to which '−q' charge is induced in the inner surface of Y and the outer surface is earthed as shown in figure.
If 'σ' is the surface charge density, then the strength of electric field between the plates is given by,



fig

$$\boxed{E = \frac{\sigma}{\epsilon}} \text{—(i)}$$

& Potential difference between two plates is,

$V = Ed$ [∵ $E = \frac{V}{d}$]

or, $V = \frac{\sigma}{\epsilon} \times d$ [From (i)]

or, $V = \frac{q}{A} \times \frac{d}{\epsilon}$ [∵ $\sigma = \frac{q}{A}$]

or, $\frac{q}{V} = \frac{A\epsilon}{d}$ [where, $C = \frac{q}{V}$ is the capacitance of parallel plate capacitor]

or, $$\boxed{C = \frac{A\epsilon}{d}}$$

$$\boxed{C = \frac{K\epsilon_0 A}{d}}$$

K → dielectric constant.

Factors on which capacitance of parallel plate capacitor depends upon:

(i) Area of plate $[C \propto A]$

(ii) Distance between the plates $[C \propto \frac{1}{d}]$

(iii) Dielectric medium (ϵ) $[C \propto \epsilon]$

76 Loss of energy by joining two capacitors

If two capacitors are connected with the conducting wire, charge flows from higher potential capacitor to lower potential one. In this process work should be done to transfer the charge from one capacitor to another capacitor until both has same common potential, which makes the loss of energy in capacitor in the form of heat, light, sound, etc. However, charge remains ~~conserve~~ same.

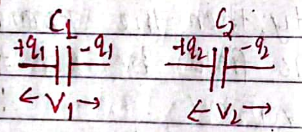


Fig: Before connection

Let, two capacitors of capacitances C_1 and C_2 be charged by the charges q_1 and q_2 to potentials V_1 and V_2 respectively.

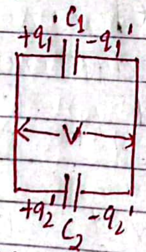


Fig: After connection

Before joining, total energy,

$$\boxed{U_1 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2} \quad \text{(i)}$$

After joining, total energy,

$$U_2 = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$= \frac{1}{2}(C_1 + C_2) V^2 \quad \text{[where, } V \rightarrow \text{common potential]}$$

Note,
Common potential (V) = $\frac{\text{Total charge}}{\text{Total capacitance}}$

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$[\because q_1 = C_1 V_1$ & $q_2 = C_2 V_2]$

$$U_2 = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right)^2$$

$$\therefore \boxed{U_2 = \frac{1}{2}\frac{(C_1 V_1 + C_2 V_2)^2}{C_1 + C_2}} \quad \text{(ii)}$$

Loss of energy, $\Delta U = U_1 - U_2$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2}\frac{(C_1 V_1 + C_2 V_2)^2}{C_1 + C_2}$$

$$= \frac{1}{2}\left[\frac{C_1^2 V_1^2 + C_2^2 V_2^2 + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2 - 2C_1 C_2 V_1 V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1 C_2 (V_1^2 + V_2^2 - 2V_1 V_2)}{C_1 + C_2}$$

$$\therefore \boxed{\Delta U = \frac{1}{2}\frac{C_1 C_2}{(C_1 + C_2)}(V_1 - V_2)^2}$$

The terms, $(V_1 - V_2)^2$, C_1 and C_2 in R.H.S. of the equation are always positive. So ΔU is always positive which means there is always loss of energy when two capacitors of different potentials are joined.

Optics
Dispersion

11. Condition for achoromatism in two thin lenses in contact.

→ Due to the prismatic behaviour of lens, when a beam of white light parallel with principle axis is incident on it, it splits up into its different constituent colours.
This inability of a lens to focus all colours of light at a single point is called the chromatic aberration. It is measured by the difference in focal length between red and violet colours.
i.e.
Chromatic aberration $= f_r - f_v = \omega f$

$\omega \to$ Dispersive power
$f \to$ focal length for mean colour of light.

→ The combination of two thin lenses in which their combination is free from chromatic aberration is called the achromatic aberration of lenses.

→ Let us consider two thin lenses L and L' of dispersive power ω and ω' respectively placed in contact with each other. Let μ_v, μ and μ_r are the refractive indices of L for violet, mean and red colour respectively, and f_v, f and f_r be focal length of respective colours. Similarly μ_v', μ', μ_r'; f_v', f', f_r' be corresponding quantities of L'.
For lens L, focal length of mean colour is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or, $$\boxed{\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f(\mu-1)}} \quad \text{(i)}$$ where R_1 and R_2 are radii of curvature of two surfaces.

Focal length of lens L for violet colour is

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or, $$\boxed{\frac{1}{f_v} = \frac{(\mu_v - 1)}{f(\mu - 1)}} \quad \text{(ii)}$$

Similarly, ~~of~~ lens L' for violet colour is

$$\boxed{\frac{1}{f_v'} = \frac{(\mu_v' - 1)}{f'(\mu' - 1)}} \quad \text{(iii)}$$

If F_v is the combined focal length of two lenses for violet colour, then

$$\frac{1}{F_v} = \frac{1}{f_v} + \frac{1}{f_v'}$$

or, $$\boxed{\frac{1}{F_v} = \frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu_v' - 1}{f'(\mu' - 1)}} \quad \text{(iv)}$$

In the same way for red colour,

$$\boxed{\frac{1}{F_r} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{(\mu_r' - 1)}{f'(\mu' - 1)}} \quad \text{(v)}$$

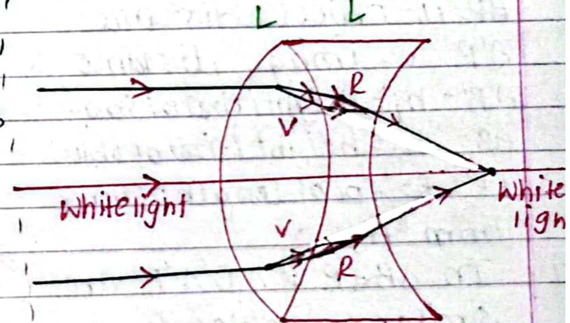


Fig: Achromatic combination of lenses

For achromatic combination, we have

$$F_v = F_r$$

or, $$\frac{1}{F_v} = \frac{1}{F_r}$$

or, $$\frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu_v' - 1}{f'(\mu' - 1)} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu_r' - 1}{f'(\mu' - 1)}$$

or, $$\frac{\mu_v - 1 - \mu_r + 1}{f(\mu - 1)} = \frac{\mu_r' - 1 - \mu_v' + 1}{f'(\mu' - 1)}$$

or, $$\frac{\mu_v - \mu_r}{f(\mu - 1)} = -\frac{(\mu_v' - \mu_r')}{f'(\mu' - 1)}$$

or, $$\frac{\omega}{f} = -\frac{\omega'}{f'}$$ where $\frac{\mu_v - \mu_r}{\mu - 1} = \omega$ & $\frac{\mu_v' - \mu_r'}{\mu' - 1} = \omega'$

$$\therefore \boxed{\frac{\omega}{f} + \frac{\omega'}{f'} = 0}$$

This is the condition for achromatic ~~aberration~~ combination of two lenses.

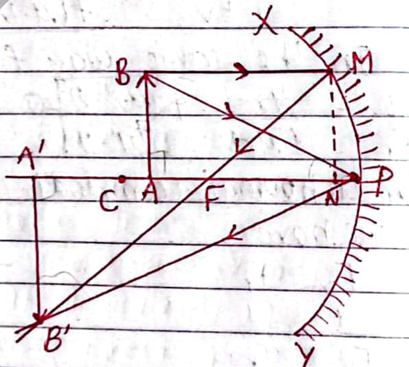
78. Mirror formula for Concave Mirror:

(i) When Real Image is formed.

A real and inverted image is formed when an object is placed on the principle axis beyond the principle focus (F) of a concave mirror.

Consider a concave mirror of aperture XY and pole P. A real object AB is placed at distance AP beyond the focus (F) of the mirror. As the object is placed beyond F of concave mirror, a real and inverted image A'B' will be formed at a same side of object. A normal MN is drawn from reflecting point M of the mirror.

Here,
AP = u = object distance
A'P = v = image distance.
A'B' = h_i = height (size) of image
AB = h_o = height (size) of object
FP = f = focal length of mirror.
From fig.



Real image formed by concave mirror.

(i) In ΔBAP & ΔB'A'P are similar triangles. So,

$$\frac{A'B'}{AB} = \frac{A'P}{AP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{v}{u}} \quad \text{—(i)}$$

(ii) In ΔB'A'F and ΔMNF are also similar triangles. So,

$$\frac{A'B'}{MN} = \frac{A'F}{FN}$$

Here, MN = AB and FN ≈ FP, we have

$$\frac{A'B'}{AB} = \frac{A'P - FP}{FP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{v-f}{f}} \quad \text{—(ii)}$$

From (i) & (ii),

$$\frac{v}{u} = \frac{v-f}{f}$$

or, $vf = vu - fu$

or, $vu = vf + fu$

Dividing by uvf on both sides.

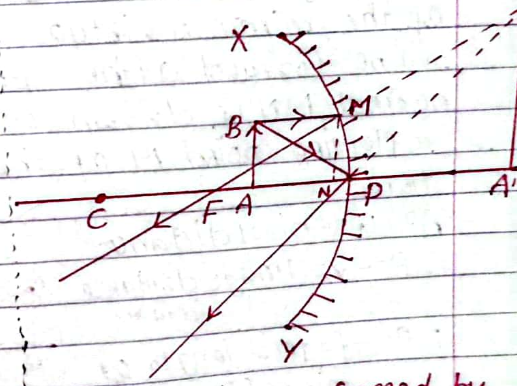
$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

This is required mirror formula.

(ii) When virtual image is formed.

Consider a concave mirror of aperture XY and pole P. A real object AB is placed at distance AP in between the focus (F) and pole (P) of the mirror. As the object is placed nearer than F of concave mirror, a virtual and erect image A'B' will be formed at the opposite side of object. A normal MN is drawn on the principle axis from reflecting point M of the mirror.

Here,
AP = u = object distance
A'P = −v = image distance (virtual)
A'B' = h_i = height (size) of image
AB = h_o = height (size) of object
FP = f = focal length of mirror.



Virtual image formed by a concave mirror.

i) In ΔBAP and ΔB'A'P are similar triangles. So,

$$\frac{A'B'}{AB} = \frac{A'P}{AP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{-v}{u}} \quad \text{—(i)}$$

(ii) In $\Delta B'A'F$ and ΔMNF are also similar triangles. So,

$$\frac{A'B'}{MN} = \frac{A'F}{FN}$$

Here, MN = AB and $FN \approx FP$, we have

$$\frac{A'B'}{AB} = \frac{A'P + FP}{FP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{-v+f}{f} = \frac{-(v-f)}{f}} \quad \text{--- (ii)}$$

Equating (i) and (ii), we get

$$\frac{-v}{u} = \frac{-(v-f)}{f}$$

or, $vf = vu - uf$

or, $vu = vf + uf$

Dividing uvf on both sides, we get

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

This is the required mirror formula.

79. Mirror formula for Convex Mirror.

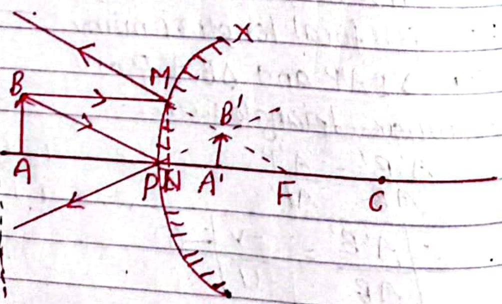
Consider a convex mirror of aperture XY and pole P. A real object AB is placed at distance AP in front of the mirror. A virtual and erect image A'B' will be formed at the opposite side of object. A normal MN is drawn on the principal axis from reflecting point M of the mirror.

Here,

AP = u = object distance

A'P = -v = image distance (virtual)

FP = -f = focal length of mirror (convex)



Virtual image formed by a

From figure,

(i) In ΔBAP and $\Delta B'A'P$ are similar triangles. So,

$$\frac{A'B'}{AB} = \frac{A'P}{AP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{-v}{u}} \quad \text{--- (i)}$$

(ii) In $\Delta B'A'F$ and ΔMNF are also similar triangles. So,

$$\frac{A'B'}{MN} = \frac{A'F}{FN}$$

Here, MN = AB and $FN \approx FP$, we have

$$\frac{A'B'}{AB} = \frac{FP - A'P}{FP}$$

$$\therefore \boxed{\frac{A'B'}{AB} = \frac{-f-(-v)}{-f} = \frac{-(v-f)}{f}} \quad \text{--- (ii)}$$

Equation (i) & (ii),

$$\frac{-v}{u} = \frac{-(v-f)}{f}$$

or, $vf = uv - uf$

or, $uv = vf + uf$

Dividing uvf on both sides, we get,

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

This is the required mirror formula.

80. Method of charging a body positively by induction.

Electro static induction:

The temporary electrification of a conductor by bringing a charged body close to it is called electrostatic induction

→ A body can be charged positively by the following ~~method:~~ ways:

Step I: An uncharged conducting rod PQ is taken on an insulating stand.

Step II: A negatively charged body is brought near to the neutral rod (but not touched). Then due to electrostatic induction the nearer end P acquires bound positive charge and farther end Q acquires free ~~pos~~ negative charge.

Step III: The rod PQ is earthed with the help of metal wire. In this case negative charge flows to the earth, but the positive bound charges remain on the conductor.

Step IV: Now, the conducting metal wire is disconnected from the rod PQ and negatively charged inducing body is taken away from the rod. Then the positive charges spreads uniformly over the rod and it becomes positively charged.

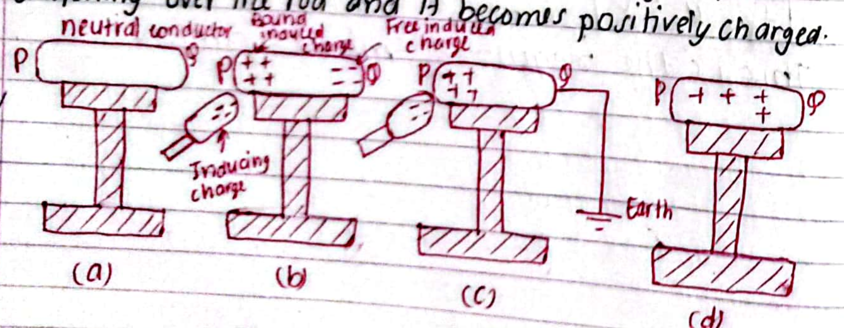


Fig: Charging a body positively by induction.